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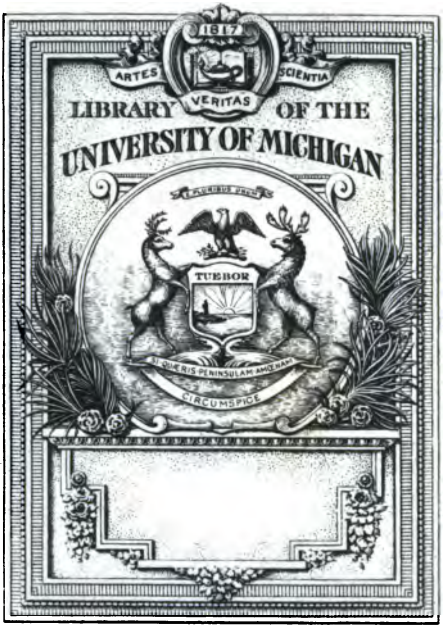

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CONTAINING
Plain, Eaſy, and Conciſe *RULES* in that
MYSTERIOUS SCIENCE.

Exemplify'd by Variety of PROBLEMS, both in
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their Equations in Numbers.

By a Univerſal Method of *Extracting the Roots* out of
all Kind of *Equations*, either Pure or Adfected, with
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of *Annuities*, and *Leaſes for Lives*, &c.

By JOHN WARD, of *Cheſter*;
Formerly Chief Surveyor and Gauger-General in the Exciſe.

The Second Edition, Carefully Corrected;
And Improved with large ADDITIONS, *throughout the Whole,*
By the AUTHOR.

LONDON,
Printed for DAN. BROWNE *Jun*r. at the *Black-Swan* without *Temple-Bar*. M.DCC.XXIV.

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THE PREFACE.

IT may be expected I ſhould here give the Reader ſome Account of the enſuing Treatiſe, more than is done in the Title-Page; and of the Motives that induced me to publiſh it. I think it needleſs to enumerate the particular Advantages that accrue to Mankind from the Effects of Mathematical Learning in general; therefore ſhall only give the Reaſons for my firſt publiſhing this Compendium, (Anno 1695.)

Having many Years ago made ſome progreſs in thoſe Studies, eſpecially in Algebra, *the which I perceived to be ſo very uſeful in the whole Series of the Mathematicks; that*

I concluded it might juſtly be called the Kernel or Seed (as it were) of thoſe Studies; and therefore one may adventure to ſay, That whoever makes himſelf once Maſter of that Science, it will give him ſuch a Habit of cloſe Thinking, and true Reaſoning in that ſort of Learning, that he need not be afraid to venture upon any Branch thereof, becauſe 'tis a Science by which the moſt Abſtruſe or Difficult Problems, either in Numbers, or Geometry, are not only reſolved, but demonſtrated: And therefore 'tis ſometimes called Arithmetick in Specie, (*as by Dr.* Wallis;) *and ſometimes 'tis call'd* Modern Geometry; *particularly the Learned and Ingenious Mathematician Dr.* Halley, *the Royal Aſtronomer, writes thus of it:*

> ‘*The Excellency of the Modern Geometry (meaning* Algebra) *is in nothing more evident (ſaith he) than in thoſe full and adequate Solutions it gives to Problems; repreſenting all the poſſible Caſes at one View, and in one General Theorem many times comprehending whole Sciences; which being deduced into Propoſitions at length, and demonſtrated after the manner of the Antients, might very well become the Subjects of large Treatiſes: For whatſoever Theorem ſolves the moſt complicated Problem of the Kind, does with a due Reduction reach all the Subordinate Caſes.*

Here you have the Opinion of one of the moſt Eminent Mathematicians in Europe *concerning* Algebra; *and as a Confirmation thereof, he hath given an undeniable Inſtance in the Doctrine of Dioptricks, for finding the Foci of Optick Glaſſes univerſally.* Vide Phil. Tranſ. *Numb.* 205.

Theſe Truths have often cauſed me to wonder that ſo valuable a Science was not more taken notice of, eſpecially amongſt ſuch Perſons, as had both a Genius and Fancy that way. But when I conſider'd that there were (in thoſe days) but few Books in Engliſh *which treated on that Subject; and thoſe were either in large Volumes (as* Kerſey's *Works in Folio;) or if otherwiſe, they were wrote in ambiguous Terms, hard to be underſtood, (as Mr.* Oughtred's *Key to the Mathematicks:) So that many who had a deſire of gaining ſome knowledge in that Art, perhaps either wanted Time, or other Conveniencies ſuitable to the peruſal of ſuch Authors: And not only ſo, but I've heard ſeveral Ingenious Perſons (pretty well verſ'd both in Geometry and Aſtronomy) ſay they wholly declin'd medling with* Algebra, *becauſe of the Difficulty of Reſolving Affected Equations above the Third Power into Numbers; there never having been any Method preſcribed for performing that difficult Part, until Mr.* Joſeph Raphſon *publiſh'd a Tract in* Latin (1690) *by* *by the Title of* Analyſis Univerſalis; *which did in a great meaſure anſwer what was before wanting, but not without ſome Difficulties.*

Theſe Conſiderations prevailed with me to think, that if there were ſome ſmall Treatiſe in Engliſh, *which began with the firſt Rudiments of* Algebra, *and proceeded gradually on through the ſeveral parts of it, plainly ſhewing how to form, and bring any Problem to an Equation, and then to reſolve that Equation into Numbers, tho' it were never ſo high or adfected, with Eaſe and Expedition; ſuch a Treatiſe muſt needs be very acceptable to all thoſe who had a deſire of being acquainted with that Myſterious Science.*

For theſe Reaſons, join'd with a Willingneſs to promote Mathematical Learning, I did (according to the beſt of my Abilities) *compoſe this* Compendium; *contracting it into as narrow a Compaſs as poſſibly I could, both for the Conveniency of Price, and that it might be a Pocket Companion. How well it hath anſwer'd my Intentions, doth fully appear from the Reception it hath already found in the World: For tho' there were above a Thouſand of them printed off in the firſt Impreſſion, yet in a few Years it became ſo ſcarce, that it was ſold for more than double the firſt Price. The which, and the frequent*

quent Demands for it, induced the Bookſeller to deſire it might be reprinted: And in order to that, he prevailed with me to reviſe and prepare it for a new Impreſſion; which accordingly I have done, and not only ſo, but I have made ſeveral large Additions to it.

Firſt, There is prefixed (by way of Introduction) that Uſeful Part of Numbers called Decimal Arithmetick, *which was omitted and much wanted in the firſt Impreſſion.*

There is alſo added a whole Chapter of managing ſuch Surd Quantities as may (and often do) *ariſe in bringing Problems to an Equation.*

I have added ſeveral pleaſant Arithmetical Queſtions, &c. *But above all,*

There is added an Eaſy Method of finding the Circle's Periphery and Area; with the like for finding the Superficies and Solidity of any Sphere or Globe; all their Proportions being traced from the Original or firſt Principles of Geometry, proved by a very plain and eaſy Demonſtration.

There are likewiſe added two different Methods of making Natural Sines and Tangents, demonſtrated with Examples of their Calculations at large.

Theſe, with ſeveral other Improvements, the Reader will find in the Algebra *Part of this Treatiſe.*

To the firſt Impreſſion of that Part, there was annexed a ſhort Appendix concerning Intereſt and Annuities, to be perform'd by Algebraick Theorems only: which was rather to ſhew what might be done by the Pen, without the help of a great many Tables of ſeveral Rates, than for common Practice

And therefore I reduced thoſe Theorems into Practical Rules expreſs'd in Words at length, and printed them in a ſmall Tract (Anno 1710) *by the Title of* Clavis Uſuræ, *or a* Key to Intereſt, both Simple and Compound, *&c.*

That Tract the Bookſeller deſired might be join'd to the Algebra *Part of this Treatiſe, inſtead of the above-mention'd Appendix.*

To this are added ſeveral Theorems with their Demonſtrations; and in order to render the whole Buſineſs of Intereſt and Annuities plain and eaſy to be underſtood, 'tis all expreſs'd in Words at length, with Variety of Examples in all their Caſes, both as to different Rates of Intereſt, and Times of Payments, with all the Operations at large. For altho' Rules are never ſo well expreſs'd in Words (according to the Author's Senſe) *yet there may (and often does) ariſe ſome ſeeming Difficulties in them, which Examples may help to explain and render eaſy.*

Thus you have an Account of the enſuing Pages, which I hope will be found Uſeful, according as it is intended and deſired

Cheſter,
July 1723.

By J. W.

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THE

INTRODUCTION:

Conſiſting of

Decimal Arithmetick.

I HERE take for granted, that the Reader is well acquainted with Vulgar Arithmetick, eſpecially with the four common Rules, *viz.* Addition, Subſtraction, Multiplication, and Diviſion of Whole Numbers.

But becauſe the Calculations in this Treatiſe, *viz.* both thoſe in the Solution of Algebraick Equations, as alſo thoſe in Computing of Intereſt, &c. require a competent Knowledge of the aboveſaid Rules in Decimal Parts, which perhaps he is not ſo well acquainted with; I have therefore thought it neceſſary in this Impreſſion to give a brief account of thoſe Rules; that ſo he may not be unprepared for what follows.

Note, The want of this was complained of in the firſt Impreſſion.

Sect. 1. Notation *of Decimals.*

Any thing which is called One, (whether it be *Coin*, *Weight*, *Meaſure*, or *Time*, &c.) as one *Foot*, one *Yard*, one *Pound*, one *Shilling*, one *Year*, or one *Day*, &c. we conceive it to be *divided* into ten equal Parts, and every one of thoſe Parts, are ſuppoſed to be ſub-divided into other ten equal Parts; and ſo on by a *Decimal Diviſion* (*viz.* by Tens) *ad infinitum.*

The *Unit*, or 1, being thus divided by Imagination into 10 . 100 . 1000 . or 10000 &c. equal Parts at pleaſure, any *Number* of thoſe Parts may be as eaſily expreſs'd and ſet down as whole *Numbers*, they being only diſtinguiſhed and known from whole Numbers, by a *Comma*, or *Point*, as in the following Table:

Whole Numbers.							*Decimal Parts.*						
	5	4	3	2	1	0,	1	2	3	4	5	6	
&c.	*Hundreds* of *Thouſands*	*Tens* of *Thouſands*	*Thouſands.*	*Hundreds.*	*Tens.*	*Unit's Place.*	*Parts* of *Ten*, or $\frac{1}{10}$	*Parts* of a *Hundred*	*Parts* of 1000	*Parts* of 10000	*Parts* of 100000	*Parts* of 1000000	&c.

From this Table it does plainly appear,

1. That, as whole Numbers do increaſe or become greater from the *Unit*'s Place towards the Left Hand, by a ten-fold Proportion: ſo *Decimal* Parts do decreaſe or become leſs from the *Unit*'s Place towards the Right Hand, in the ſame Proportion, *viz.* by *Tens.*

2. That the *Decimal* Parts are only ſeparated and known from Whole Numbers by a *Point*, or *Comma*, and take their Denomination from their Diſtance below the Unit's Place towards the Right Hand.

Thus	0,5	is 5	parts of *Ten.*
	0,25	is 25	parts of a *Hundred.*
	0,657	is 657	parts of a *Thouſand*, &c.

3. *Cyphers* being annexed to *Decimal* Parts, alter not their *Values*; that is, thoſe *Cyphers* do neither increaſe nor decreaſe the Value of the Parts they are annexed to.

Thus 0,5 0,50 0,500 0,5000 *&c.* are each of them but five tenth parts of a Unit or 1.

4. But *Cyphers* prefixed before *Decimal* Parts, do decreaſe their *Value*, by removing them further from the *Comma*, or *Unit's* place.

Thus { 0,5 is 5 parts of *Ten.*
0,05 is 5 parts of a *Hundred.*
0,005 is 5 parts of a *Thouſand*, &c.

Conſequently the true *Value* of all *Decimal* Parts is eaſily known by their diſtance from the *Unit's* place (as above:) Which being well underſtood, the following *Rules* will be found very eaſy.

Sect. 2. Addition *and* Subſtraction *of* *Decimals.*

Having ſet down all the propoſed Numbers in their reſpective places, *viz.* every *Figure*, as well of the *Decimal* Parts, as of the Whole *Numbers*, directly underneath thoſe of the ſame *Value*, (or Name) which may be very eaſily done, if the *Comma's*, or ſeparating *Points*, are carefully placed directly one under another:

Then { *Add*, or *Subſtract* them as if they were all whole Numbers; and from their Sum, or Difference, cut off, by the ſeparating Comma, ſo many places of Decimal Parts, as there are in any of the given Numbers.

Examples in Addition.

Let it be required to find the *Sum* of 43,25 and 2,45 and 29,9, and 7,054. Theſe Numbers being rightly ſet down will ſtand

Thus	43,25	Again	378,0009
	2,45		59,025 } Add
	29,9		470,9
	7,054		92,0741
Sum	82,654	Sum	1000,0000

Examples in Subſtraction.

From	753,25	From	75,094	From	543,000
Take	469,25	Take	68,596	Take	267,546
Rem.	284,00		6,498		275,454

Note, *Addition* and *Subſtraction* do mutually prove each other, as in whole *Numbers*; of which I ſuppoſe it needleſs to inſert Examples.

Sect. 3. Multiplication *of Decimals*.

Whether the *Factors* (*viz.* the Numbers) propoſed to be *multiplied* together, are either all *Decimal* Parts, or *Decimals* joined to whole *Numbers*, *multiply* them together as if they were all whole Numbers, and for the true *Value* of their Product, obſerve this *Rule*.

Rule { *Cut off* (*viz. ſeparate*) *with a Comma, ſo many places of Decimal parts in the Product as there are in both the Factors counted together.*

Examples.

Suppoſe it were required to *multiply* 21,3 with 4,56

The given Factors { 21,3 / 4,56 — Or { 4,56 / 21,3

21,3	4,56
4,56	21,3
1278	1368
1065	456
852	912
Product 97,128	97,128

Again, Let it be required to *multiply* 4573 *into* 7,546; And 36,4078 *into* 549,3. The Work may ſtand

Thus 4573 }	*Multiply* { 36,4078
7,546 }	549,3
27438	1092234
18292	3276702
22865	1456312
32011	1820390
34507,858 *Products*	19998,80454

Note, It ſometimes happens, that in *Multiplying* Parts with Parts only, there will not be ſo many *Figures* in their Product as there ought to be Places of Parts by the *Rule*: In that Caſe you muſt ſupply the Defect by *prefixing Cyphers* to their Product. As in theſe following

Examples.

0,2365	*Multiply*	0,03472
0,2435		0,02364
11825		13888
7095		20832
9460		10416
4730		6944
0,05758775	*Products*	0,0008207808

Sect. 4 Diviſion of *Decimals.*

Diviſion of *Decimals* is perform'd in all reſpects like that of whole *Numbers*: And for diſcovering the true *Value* of the *Quotient*, obſerve this General *Rule*.

Rule { *The places of Decimal parts in the Diviſor and Quotient being counted together, muſt always be equal in number with thoſe in the Dividend.*

This General Rule admits of four *Caſes*.

Caſe 1. When the Places of *Decimal* parts in the *Diviſor* and *Dividend* are equal in number, then the *Quotient* will be all whole Numbers: As in the two following Examples.

Dividend.

Diviſor 7,54) 7140,38 (947 the *Quotient.*
6786
3543
3016
5278
5278
(0)

Again, If it were required to divide 2,6925 by 0,0075

Dividend.

Diviſor 0,0075) 2,6925 (359 *Quotient.*

Caſe 2. When the Number of Places of Decimal Parts in the Dividend, exceed thoſe in the Diviſor, *cut off* the Exceſs of Decimal Parts in the Quotient.

Examples.

7,54) 71,4038 (9,47   And 0,75) 2,6925 (3,59

Caſe 3. When there are not ſo many places of Decimal parts in the Dividend, as there are in the Diviſor, then annex Cyphers to the Dividend, to make them equal, and the Quotient will be all whole Numbers: As in *Caſe* 1.

Examples.

Let us ſuppoſe it were required to divide 1565,7 by 3,684: And 3615 by 5,784; the Work muſt ſtand thus,

3,684)1565,700(425.   And 5,784)3615,000(625.

Caſe 4. If, after the Diviſion is finiſhed, there are not ſo many Figures in the Quotient, as there ought to be Places of Decimal Parts, by the General *Rule*, you muſt then ſupply their Defect, by *prefixing* Cyphers before the Quotient Figures.

Examples.

Let it be required to divide 3,5532 by 756

```
756 ) 3,5532 ( 47
      3024
      ----
       5292
       5292
       ----
        (0)
```

Here the Diviſion being finiſhed, there are but *two* Figures in the Quotient, whereas there ought to be four places of Decimal Parts, by the general *Rule*; therefore two Cyphers muſt be prefix'd before the Quotient Figures, thus 0,0047

That is, 756) 3,5532 (0,0047 is the true Quotient required.

Suppoſe it were required to divide 0,0007475 by 0,575; *viz.* all Decimal Parts: Then
It will be 0,575) 0,0007475 (0,0013 the Quotient, *&c.*

Note, *Multiplication* and *Diviſion* do mutually prove the Truth of each other: That is, if in *Multiplication* you divide the Product by either of the Factors, the Quotient will be the other Factor, if the Work be true.

So in *Diviſion*, if you multiply the Quotient and the Diviſor together, their Product (*being added to what remain'd after Diviſion*) will be the Dividend, if that Work be true.

Sect. 5. *To* Reduce (*or rather* Change) *Vulgar Fractions into Decimal Parts, and the contrary.*

Rule. *Annex Cyphers to the Numerator of the given Fraction; then divide it by the Denominator, and the Quotient will be the Decimal parts ſought.*

Examples.

Suppoſe it were required to *reduce* $\frac{3}{4}$ into Decimal parts: Then it will
be, 4) 3,00 (0,75 the Decimal parts required;
And $\frac{1}{2}$ is 0,5 Thus 2) 1,0 (0,5 the Decimal;
Or $\frac{1}{4}$ is 0,25 Thus 4) 1,00(0,25 the Decimal.

And thus may the Decimal parts *equivalent* to any known part, or parts of *Coin*, *Weights*, *Meaſures*, or *Time*, &c. be eaſily found, if you firſt reduce the given parts of the *Coin*, or *Time*, &c. into a *Vulgar Fraction* whoſe Denominator is the Number of thoſe known parts, contained in the Integer, and the given parts its Numerator.

Examples in Coin.

Suppoſe it were required to find the Decimal Parts of a Pound Sterling, equal to 17 *s.* 6 *d.*

Firſt, becauſe 20 *s.* makes one Pound, therefore 17 *s.* is $\frac{17}{20}$ of a Pound; and becauſe there's 240 Pence in a Pound, therefore 6 *d.* is $\frac{6}{240}$ of a Pound.

Then 20) 17,00 (0,85 is the Decimal of 17 *s.*
And 240) 6,000 (0,025 is the Decimal of 6 *d.*
Conſequently their Sum 0,875 will be the Decimal parts equivalent to 17 *s.* 6 *d.* as was required, *&c.*

Example in Time.

Let it be required to find the Decimal parts equal to 126 Days.

Firſt, becauſe 365 Days make a common Year, therefore 126 Days is $\frac{126}{365}$ parts of a Year.

Then 365) 126,0000 (0,3452 will be the Decimal parts equivalent to 126 Days, &c.

The like is to be underſtood in *reducing* or *changing* the known parts of any propoſed Integer into Decimal parts, equal to thoſe known parts; or at leaſt ſo near the Truth, as may be thought neceſſary to approach.

Now the contrary Operations to theſe, *viz.* to find the *Value* of any given Decimal parts of *Coin* or *Time*, &c. is only the *Converſe* of the Work above, and may be performed by this following Rule.

Rule { *Multiply the given Decimal parts with ſuch a Number of Units as are contained in the Integer to which the given Decimal does belong, and the Product will be the Value of the ſaid Decimal.*

Examples.

1. Let 25,7875 ſignify 25 *l.* and the Decimal parts of 1 *l.* How many Shillings and Pence do theſe Decimal parts denote? That is, How many Shillings and Pence are equal to 0,7875 Decimal parts of a Pound *Sterling*.

Firſt,	25,7875	
Multiplied with	20	the Shillings in 1 *l.*
Gives Shillings	15,7500	and the Decimals of 1 *s.*
Multiply with	12	the Pence in 1 *s.*
	150	
	75	
Gives Pence	9,0000	Hence the Anſwer is,

That 25 *l.* 15 *s.* 9 *d.* is the ſame with 25,7875 Decimals of *Coin*.

2. Suppoſe the ſame 25,7875 to ſignify 25 Years, and the Decimal parts of one Year; How many *Days*, *Hours*, &c. would thoſe Decimal parts denote?

Anſwer 25 *Years*, 287 *Days*, 10 *Hours*, and 30 *Min.*

Thus

Thus	25,7875	
Multiplied with	365	the Days in One Year.
	39375	
	47250	
	23625	
Gives Days	287,4375	and Decimal Parts of 1 Day,
Multiply with	24	the Hours in 1 Day.
	17500	
	8750	
Gives Hours	10,5000	and 0,5 parts of 1 Hour,
Multiply with	60	the Minutes in 1 Hour.
Minutes	30,0000	

And thus may any propoſed Number of Decimal Parts be *reduced* or *changed* into the known Parts of what they repreſent, *viz.* whether they be Parts of *Coin*, *Weight*, *Meaſures*, or *Time*, *&c.* a due regard being had to the Number of Units which are contain'd in the Denomination of the thing to which the Decimal Parts belong.

PART I.

A
COMPENDIUM
OF
ALGEBRA.

CHAP. I.

Of the Nature of Algebra, *and Definition of the* Symbols, *or* Characters, *uſed therein*, &c.

I SHALL not undertake (in this ſmall Tract) to give an Hiſtorical Account of the Original, or Antiquity either of this Art, or its Name; but confine myſelf within the Limits of this Definition, that (that which is now called) *Algebra*, is an Art by (*the help of*) which the moſt difficult and abſtruſe Queſtions, either in *Arithmetick* or *Geometry*, may be reſolved, by an univerſal Method of Reaſoning, and comparing of known Quantities with thoſe that are unknown.

And this way of arguing, or comparing of Quantities with Quantities, not only diſcovers an Anſwer to the Queſtion propoſed, but at the ſame time doth raiſe (and demonſtrate) Theorems, or Rules, for the reſolving of all Queſtions in the like nature.

And from hence it is, that ſo many curious and uſeful Theorems, Canons, or Rules are firſt diſcovered, which are of excellent uſe in the Practice, but would for ever lie hid were it not for this Art.

This *Algebraick Arithmetick* is perform'd by ſubſtituting of Alphabetical Letters, to repreſent as well the known, as the unknown Quantities concern'd in any Problem, (be it Arithmetical or Geometrical) and to theſe ſubſtituted Letters are prefixed certain Symbols, Notes or Characters, by the help of which, the Quantities (in their Subſtitutes) are ſo connected and ordered, that thereby they become capable of being compared, and moulded by Addition, Subſtraction, Multiplication and Diviſion, *&c.* according to the Deſign or Nature of the Queſtion: And this is ſo done, as tho' all the Quantities concern'd therein, (as well the ſought as the given) were really known; till at laſt the given Quantities come to an Equation (or Equality) with the Quantity that's ſought, (or ſome Power thereof) that ſo it may be found. In all which various Operations and Compariſons, all the Quantities concern'd in the Queſtion, remain in their Species ſo plain and clear, that they neither burden the Memory, nor rack the Fancy; but, on the contrary, help both, by repreſenting to the Eye (at once) the Proceſs of each Operation, uſed throughout the whole Management of a Queſtion.

The Method of noting down theſe Subſtitutes in this Tract, is thus: I make choice of repreſenting the Quantity ſought (be it a Line or Number) by the ſmall (*a*,) and if more than one are ſought, I make uſe of the Vowels (*e*,) or (*y*,) *&c.* to repreſent them. And for the given Quantities, be they either Numbers, Lines, or other Magnitudes) I repreſent them by the Conſonants *b*. *c*, *d*, *f*, *g*, *&c.* and, for diſtinction, I note (as uſual) all Schemes with the Great or Capital Letters, and work all Operations with the Small Letters.

If any Quantity be taken more than once, prefix its Number; as $6a$, $5b$, $9c$, $3\frac{1}{2}d$, *&c.* theſe ſtand for a, taken ſix times, or ſix times a, five times b, *&c.* And all Numbers thus prefixed to any Quantity, are called Coefficients, (or Fellow-Factors) but if the Quantity have no prefixed Number, (or Coefficient) then ſuch Quantity ſtands for itſelf once taken, and is ſuppoſed (or underſtood) to have an Unit prefixed to it, as a is $1\,a$, or b is $1\,b$, *&c.*

The Signs made uſe of in this Tract, for the more ſhort and quick expreſſing of Words, are the moſt general now in uſe, and are thoſe following.

N. B. Note, That it is very requiſite for the young Learner of this Art to be very perfect in the knowledge of the Signs, and their Significations, before he proceeds furthere therein; that is, before he intermeddles with any of the following Rules.

Alſo I adviſe him to be very ready in one Rule, before he undertakes the next; that is, to be expert in, and underſtand *Addition*, before he meddles with *Subſtraction*; and that of *Subſtraction*, before he undertakes that of *Multiplication*, *&c.* becauſe they have each a Dependency one upon the other.

And by this Method of proceeding, he may make himſelf Maſter thereof, or at leaſt arrive to a competent knowledge therein, with eaſe, and in a very ſhort ſpace of time.

Signs.	*Names.*	*Significations.*
$+$	*More.*	The Affirmative Sign, and Sign of Addition, as $a+b$, ſignifieth that the Quantities a and b are to be added together.
$-$	*Leſs.*	The Negative Sign, and Sign of Subſtraction, as $a-b$, ſignifieth, that the Quantity b, is to be taken from that of a.

Signs.	*Names.*	*Significations.*
×	*Into.*	The Sign of Multiplication, as $a \times b$, ſignifieth that a is to be multiplied into b; but oftentimes the Sign is omitted, yet is underſtood, as ab, is $a \times b$.
÷	*By.*	The Sign of Diviſion, as $a \div b$, ſignifieth that a is to be divided by b; yet this Sign is alſo often omitted, as $\frac{a}{b}$ is $a \div b$, or thus, $b)a($
::	*Proportion.*	The Sign of Disjunct Proportion, as in the *Golden Rule*, and is thus, $a : b :: c : d$ to be read in this manner, as a, is to b, ſo is c, to d.
∺	*Continued.*	The Sign of Continued, or Geometrical Proportion; that is, as $a : b :: b : \frac{bb}{a} :: \frac{bb}{a} : \frac{bbb}{aa}$ &c.
∵	*Ergo.*	Signifying the Product of the two Extreams is equal to that of the Means, and [∵] is placed in the Margin after Proportion.
=	*Equal.*	The Sign of Equation, or equal to, as $a = b$, ſignifieth that a is equal to b, &c.
⊙	*Involve.*	The Sign of Involution, and is placed in the Margin of a Proceſs, to ſignify, that ſuch Quantity, or Equation to which it refers, is to be multiplied by itſelf, &c.

Signs.	*Names.*	*Significations.*
ww	*Evolve.*	The Sign of Evolution, and is placed in the Margin, to ſignify the contrary to Involution, *viz.* The Extraction of the Root, *&c.*
$\surd$	*Root.*	The Sign of Irrationality, or of a *Surd Root*; over which, if a Number be placed, it ſignifieth of what degree that Root is: And this Number is called the *Index*, thus

$\sqrt[2]{\ }$
$\sqrt[3]{\ }$
$\sqrt[4]{\ }$
$\sqrt[5]{\ }$
Denotes the
Square
Cube
Biquadrate
Surſolid
Root, &c.

$\surd$: Signifieth an *Univerſal Root*, or *Root of Roots.*

Signs.	*Names.*	*Significations.*
$>$	*Greater.*	The Sign of Majority, as $a > b$, ſignifieth that a is greater than b.
$<$	*Leſſer.*	The Sign of Majority reverſed, as $b < c$, ſignifieth that b is leſſer than c.
$\square$	*Square.*	Signifieth the Square of any Numbers, or Quantities, or of an Equation, as $\square\, a+b+c$, &c. is the Square of a, more b, more c.

The chief of theſe Signs or Characters, wherewith the ſubſtituted Quantities (or Letters) are connected, be More, and Leſs, *viz.* The Sign +, and Sign −; where *Note*, That the Sign always belongs to the Quantity that follows it, (or is to the Right Hand thereof,) and not to that which is before it. And if a Quantity hath no Sign prefix'd before it, (as generally the leading Quantity hath not) it's then underſtood to have the Sign + prefixed, as a is $+a$, or $b+d$, is $+b+d$, &c.

If ſeveral Quantities be connected, or engaged in any Queſtion, they may ſtand in any Order at pleaſure; as $a+b-d$, may ſtand thus, $a-d+b$; or thus, $-d+a+b$, &c. being ſtill the ſame, tho' differently placed; and theſe are called Compound Quantities, that is, when two or more different ſingle Quantities are connected together, by their Signs, of which more in its due place.

The Method uſed in this Tract, of placing Figures, and Signs in the Margin, according to the ſeveral Steps, or Proceſs, uſed in forming of an Equation, is the ſame with that of Mr. *Branckers*, (or rather Dr. *Pells)* in his *Introduction to Algebra*, Printed 1668, (which hath of late been followed by others) and is thus to be underſtood.

The Numbers 1, 2, 3, 4, *&c.* in the ſmall Column, do ſhew the ſeveral Steps uſed in the forming, or bringing the Quantities concern'd in any Queſtion to an Equation: And the Numbers placed in the Margin, have relation to thoſe in the ſmall Column; and the Signs placed betwixt the ſaid Numbers in the Margin, do ſhew how the Quantities that ſtand againſt the like Numbers in the ſmall Column have been ordered, either by Addition, Subſtraction, or otherwiſe. And this Method of Regiſtring the ſeveral Proceſſes, is very uſeful, eſpecially in long and tedious Operations, as will more fully appear in the Practice thereof. An Example or two will help to explain the ſame better than Words.

Suppoſe it were required to ſet down the Sum of the two Quantities a and b, according to this Method.

Thus	1	a
	2	b
$1+2$	3	$a+b$

Having placed down the Quantities a and b, againſt the Figures 1, 2, in the ſmall Column, alſo againſt 3, muſt be placed $1+2$ in the Margin, which denotes that the Quantities in the firſt and ſecond Steps, are added together, and thoſe in the third Step, are their Sum.

To illuſtrate this in Numbers, ſuppoſe $a=8$, and $b=6$.

		Quantities.	Numbers.
	1	a	8
	2	b	6
$1+2$	3	$a+b$	$8+6=14$

Again, ſuppoſe it were required to ſet down the Difference of the ſame two Quantities.

Thus	1	a	8
	2	b	6
$1-2$	3	$a-b$	$8-6=2$

Again, to ſet down their Product.

Thus	1	a	8
	2	b	6
1×2	3	ab	$8 \times 6 = 48$

Theſe Definitions of the Signs, (or Notes) Method, &c. being premiſed, and pretty well underſtood by the young Learner, (for ſo they ought to be) he may then proceed to the ſeveral parts of *Algebraick Arithmetick*, viz. *Addition*, *Subſtraction*, *Multiplication*, and *Diviſion*, &c.

In the delivery of which, I ſhall begin with ſingle whole Quantities, in each diſtinct Part thereof, then proceed in the like Order with Compound Quantities, and ſo on to Fractions; treating of each ſeparately, with what plainneſs and brevity I can.

CHAP. II.

Of the ſeveral Parts of Algebraick Arithmetick, *and how the ſame are performed.*

§. 1. Addition *of Single Whole Quantities.*

Rule 1. IF the Quantities be alike, and have like Signs, add the Coefficients (or prefixed Numbers) and adjoin the Quantities with the ſame Sign.

Thus	1	a	$-3b$	$+5bc$	$-6abc$
	2	a	$-5b$	$+4bc$	$-7abc$
1+2	3	$2a$	$-8b$	$9bc$	$-13abc$

Rule 3. When the Quantities be alike, but have unlike Signs, ſubſtract the Coefficients from each other, and adjoin the Quantities with the Sign of the greater.

Thus	1	$3a$	$-5b$	$-9bd$	$10abcd$
	2	$-a$	$+7b$	$+7bd$	$-12abcd$
1+2	3	$2a$	$+2b$	$-2bd$	$-2abcd$

Rule 3. But when the Quantities be unlike, (be the Signs alike or unlike) ſet down the Quantities with their reſpective Coefficients, without altering their Signs, and thence ariſeth Compound Quantities.

Thus	1	a	a	$-7bcd$
	2	b	$-b$	$+5bdg$
1+2	3	$a+b$	$a-b$	$5bdg-7bcd$

Alſo thus	1	a	$-5a$
	2	b	$+7bc$
	3	c	-40
1+2+3	4	$a+b+c$	$7bc-5a-40$

§. 2. Subſtraction *of Single Whole Quantities.*

General Rule.

CHange (or at leaſt in your mind ſuppoſe to be changed) all the Signs of the Subtrahend; then add, or collect all the Quantities together, (as before in Addition) and that Sum will be the Remainder, or Difference ſought.

For example, Suppoſe it were required to find the Difference of theſe Quantities:

$2a$	$8b$	$-7bc$	$2ba$
a	$5b$	$-4bc$	$2ba$

According to the Rule, the ſame Quantities will ſtand, when their Signs are changed, thus:

	1	$2a$	$8b$	$-7bc$	$2ba$
	2	$-a$	$-5b$	$+4bc$	$-2ba$
1 — 2	3	a	$3b$	$-3bc$	0

But this may be done by ſuppoſing the Signs changed, without ſetting them down again.

Thus	1	$5a$	$-4bc$	$-5bd$	0
	2	$-2a$	$+5bc$	$-8bd$	$-2ba$
1 — 2	3	$7a$	$-9bc$	$+3bd$	$+2ba$

Again,	1	a	$-4b$	$+5bcd$
	2	b	$-3a$	$-7baf$
1 — 2	3	$a-b$	$+3a-4b$	$5bcd+7baf$

Thus Compound Quantities are produced by *Subſtraction*, as they were by *Addition*.

Note, That *Subſtraction* is proved, by adding the Subtrahend to the Remainder; as in Vulgar Arithmetick.

Thus	1	$7b$	$-5bc$	$-4abd$
	2	$-2b$	$+3bc$	$-3cfg$
1 — 2	3	$9b$	$-8bc$	$+3cfg-4abd$
2 + 3	4	$7b$	$-5bc$	$-4abd$

§ 3. Multiplication *of Single Whole Quantities.*

Rule. 1. If the Quantities have no Coefficients, and have like Signs, join them together, whether they be like or unlike, and to the Product prefix the Sign +

Thus	1	a	ab	$-bc$	$-ab$
	2	b	cd	$-df$	$-d$
1 × 2	3	ab	$abcd$	$+bcdf$	abd

Rule 2. If there be Coefficients, multiply them, and to their Product join the Quantities, (ſet together as before.)

Thus	1	$5a$	$7bc$	$-24bc$
	2	b	$4ad$	$-6df$
1×2	3	$5ab$	$28adbc$	$+144bcdf$

Rule 3. And when the Quantities have unlike Signs, the Operations are the ſame with the foregoing; only to the Product prefix the Sign —

Thus	1	a	$-bc$	$-7b$	$-ba$
	2	$-b$	$+cd$	$+6ad$	$+12$
1×2	3	$-ab$	$-bcd$	$-42bad$	$-12ba$

From hence it appears, that like Signs produce the Affirmative Sign +, but unlike produce the Negative Sign —.

The reaſon thereof may be thus ſtated: Suppoſe $a =$ any Affirmative Quantity, and let $-b = -2$, then is b equal to 2 leſs than nothing, (for ſo are all Negative Quantities.) Now to multiply any Number or Quantity, implies a putting together of the ſame, ſo often as is the Number in the Multiplier, as $a \times 2 = 2a$, or a, twice put together. But if the Multiplier be -2, then it implies the taking away of a, twice; and therefore the Product muſt needs be $-2a$, that is, $a \times -2 = -2a$, or $a \times -b = -ab$, as above. But to multiply $-a$, into -2, by the ſame reaſon, it will take away the Defect of a twice, which is the ſame as to ſupply it; therefore $-a \times -2 = +2a$, or $-a \times -b = ab$.

§. 4. Diviſion of *Single Whole Quantities.*

Rule 1. IF they have like Signs, and no Coefficients, and if there be like Quantities in the Dividend, as there are in the Diviſor, caſt off ſuch Quantities in both, and ſet down the other Quantities, with the Sign +.

Thus	1	ab	$abcd$	$-bcf$	$-abc$
	2	a	bc	$-bf$	$-abc$
$1 \div 2$	3	b	ad	$+c$	$+1$

N.B. Note, That when the Quantities in the Dividend and Diviſor be all the ſame, then the Quotient is an Unit; as in the laſt above.

Rule 2. But if the Quantities have Coefficients, divide the Numbers (as in common Arithmetick) and to the Quotient prefix the Quantities.

Thus	1	$6ab$	$-12abc$	$18cd$	$-8adf$
	2	$3a$	$-3ab$	$3c$	$-2adf$
1 ÷ 2	3	$2b$	$+4c$	$6d$	$+4$

Rule 3. When the Quantities have unlike Signs, the Operations are the ſame; only ſet down the Quotient with the Sign —.

Thus	1	$-ab$	$+6ab$	$+12cd$	$-8ad$
	2	$+a$	$-3a$	$-4c$	$+4ad$
1 ÷ 2	3	$-b$	$-2b$	$-3c$	$-2ad$

But when the Quantities in the Diviſor, are not found in the Dividend, ſet down both Fraction-wiſe,

Thus	1	a	$6bc$	bdf	abc
	2	b	$3d$	abf	$4ac$
1 ÷ 2	3	$\frac{a}{b}$	$\frac{2bc}{d}$	$\frac{d}{a}$	$\frac{b}{4}$

§. 5. Involution *of Single Whole Quantities.*

INvolution is performed as *Multiplication* is, (and indeed is no more but *Multiplication*, differing only in this, *Multiplication* may be performed by different Multiplicators, but *Involution* ſtill retains the ſame.) If any Quantity (or Quantities) be involved, or drawn into itſelf, and afterwards into that Product; or again, a third time, into the laſt Product, *&c.* as manifold as is the power, ſo great muſt the Number be that is uſed to expreſs it: and this Number is called the Index of the Power, and is ſet after the Sign in the Margin.

Thus

Thus	1	a	$-2a$	bc
1 ⦵ 2	2	aa	$+4aa$	$bbcc$
1 ⦵ 3	3	aaa	$-8aaa$	$bbbccc$
1 ⦵ 4	4	$aaaa$	$+16aaaa$	$bbbbcccc$
1 ⦵ 5	5	$aaaaa$	$-32aaaaa$	$bbbbbccccc$

Which may be ſhortened, by placing the Indices over the Quantity (or Quantities) according to the height of the Power; by ſuch Method the Work will ſtand thus.

	1	a	$-a$	bc	*Root,*
1 ⦵ 2	2	a^2	$+a^2$	b^2c^2	*Squares*
1 ⦵ 3	3	a^3	$-a$	b^3c^3	*Cube*
1 ⦵ 4	4	a^4	$+a^4$	b^4c^4	*Biquadratick*
1 ⦵ 5	5	a^5	$-a^5$	b^5c^5	*Surſolid,* &c.

§ 6. Evolution (*or Extracting of Roots*), *of Single Whole Quantities.*

Rule. IF the Quantity have a Root (which is eaſily diſcover'd, by comparing its Index with the Root required) divide the Index of its Power, by 2, if the Square Root be deſired; or by 3, if you require the Cube Root, &c. and the Quotient will give the Root required, dealing with the Coefficients, or prefixed Numbers, as in Vulgar Arithmetick.

Thus	1	a^2	$36bb$	$81bbcc$
1 ɯ 2	2	a	$6b$	$9bc$

	1	a^6	$8bbb$	$bbbddd$
1 ɯ 2	2	a^3	$\sqrt{8bbb}$	$\sqrt{bbbddd}$
1 ɯ 3	3	a^2	$2b$	bd

But if either out of the Indices, or out of the Co-efficients, no exact Root can be extracted, according to the Root required; then prefix before it the Sign of the required Root: and thence will ariſe a *Surd Quantity,* as in the Square Root of the two laſt Examples above; and ſuch are ſome of theſe following,

	1	a^6	$-\ 32a^5$	ab	$216d$
1 ω 2	2	a^3	$+\sqrt[2]{32a^5}$	$\sqrt[2]{ab}$	$\sqrt[2]{216d^3}$
1 ω 3	3	a^2	$-\sqrt[3]{32a^5}$	$\sqrt[3]{ab}$	$6d$
1 ω 4	4	$\sqrt[4]{a^6}$	$+\sqrt[4]{32a^5}$	$\sqrt[4]{ab}$	$\sqrt[4]{216d^3}$
1 ω 5	5	$\sqrt[5]{a^6}$	$-\ 2a$	$\sqrt[5]{ab}$	$\sqrt[5]{216d^3}$

Thus I have gone through the ſix Parts of *Algebraick Arithmetick*, in ſingle whole Quantities, which being well conſidered, (and rightly underſtood) the following will be very eaſy; *viz.* Compound Quantities.

CHAP. III.

The Parts of Algebraick Arithmetick *in Compound Quantities.*

§. 1. ADdition of Compound Quantities is perform'd in the ſame Manner, and by the ſame Rules as ſingle Quantities are; *viz.* by putting together like Quantities, reſpect being had to the Signs: and in caſe the Quantities be not alike, connect them together by their Signs, as in the laſt caſe of Single Quantities.

Thus	1	$a + e$	$a + e$	$+ 3a - b + 3c$
	2	$a + e$	$a - e$	$- 3a + b - 2c$
1 + 2	3	$2a + 2e$	$2a$	c

Again,	1	$a - b + c - 7$	$- 4bc + bcd$
	2	$a - b - c + 3$	$+ 2bc + 3bcd$
	3	$- a + 2b + d + 6$	$+ 4bc - 2bcd$
1 + 2 + 3	4	$a + d + 2$	$2bc + 2bcd$

Again,	1	$aa + 2ab + bb$	$a^3 + 3bba + bbb$
	2	$-4ab$	$-6bba$
$1 + 2$	3	$aa - 2ab + bb$	$a^3 - 3bba + bbb$

§. 2. *Subſtraction* in Compound Quantities, is in all reſpects like that of Single Quantities, care being taken of their Signs as before.

Thus	1	$a + b$	$2a + b + 2c$	$4a + 3b - 75$
	2	$a - b$	$a + b + c$	$-a + 4b + 35$
$1 - 2$	3	$2b$	$a + c$	$5a - b - 110$

Again,	1	$aa + 2ab + bb$	c
	2	$+4ab$	$3a - b - 3c$
$1 - 2$	3	$aa - 2ab + bb$	$-3a + b + 4c$

§. 3. *Multiplication* of Compound Quantities is the ſame with that of Single Quantities; for the Product of each Member of the Multiplier, into all the Members of the Multiplicand, (reſpect had to the Signs) is the Product.

Thus,	1	$3a + 5d$	$2b + c - 4d + f$
	2	b	$2bc$
1×2	3	$3ab + 5db$	$4bbc + 2bcc - 8cbd + 2bcf$

Again at large,

	1	$a - b + c$	
	2	$b - d$	
$1 \times b$	3	$ab - bb + bc$	Product by b
$1 \times d$	4	$-ad + bd - dc$	Product by $-d$
1×2	5	$ab - bb + bc - ad + bd - dc$	

Again,

	1	$a + d + f - 3$	
	2	$b - c$	
		$ab + db + fb - 3b$	Product by $+b$
		$-ac - dc - fc + 3c$,	Product by $-c$
1×2	3	$ab + db + fb - 3b - ac - dc - fc + 3c$	

§.4.

§. 4. *Diviſion* of Compound Quantities, is perform'd like that of Single; and if any Doubts ſhould ariſe therein, they are eaſily reconciled, by conſidering, That *Diviſion* diſſolveth what *Multiplication* puts together; and therefore to be perform'd by contrary Operations.

Thus			
	1	$3ab + 5db$	$2bcf + 4bbc + 2bcc - 8bcd$
	2	b	$2bc$
$1 \div 2$	3	$3a + 5d$	$f + 2b + c - 4d$

Thus the Product of the Second Example in *Multiplication*, divided by $2bc$, gives $2b + c - 4d + f$.

Examples of Diviſion *at large.*

Suppoſe the Product of the laſt Example in *Multiplication* were given to be divided by $b - c$, the Operation will be thus perform'd.

	1	$ab - ac + db - dc + bf - fc - 3b + 3c$	
	2	$b - c$	
$2 \times a$	3	$ab - ac$	$(+a$
$1 - 3$	4	$+db - dc$	$(+d$
$2 \times d$	5	$+db - dc$	
$1 - 5$	6	$+bf - fc$	$(+f$
$2 \times f$	7	$+bf - fc$	
$1 - 7$	8	$-3b + 3c$	$(-3$
2×3	9	$-3b + 3c$	
$1 - 9$	10	0	
$1 \div 2$	11	$a + d + f - 3$. Each Quotient collected.	

Oftentimes the whole Proceſs is omitted, only ſet down with the Sign $\div$ Thus $aa - bb \div a$, or $b + a)\, aa - bb\, ($ When the Dividend will not be juſt wrought off without a Remainder, in ſuch Caſes, ſet all down (like the Terms of a Vulgar Fraction) as in the laſt Example of Single Quantities.

Thus			
	1	$ab + cb$	$aa + 2ab + cc$
	2	$d + f$	$a + 2b$
$1 \div 2$	3	$\frac{ab + cb}{d + f}$	$a + \frac{cc}{a + 2b}$

§. 5. *Involution* of Compound whole Quantities, is performed in all reſpects like *Multiplication.*

Example. Suppoſe the Square, Cube, or any other Power were required, from the Compound Root $a+b$; or $a-b$, *viz.* a Binomial, or Reſidual Root.

		Binomial.	Reſidual.
	1	$a+b$	$a-b$
		$a+b$	$a-b$
1 × a	2	$aa+ab$	$aa-ab$
1 × b	3	$+ab+bb$	$-ab+bb$
1 ⊙ 2	4	$aa+2ab+bb$	$aa-2ab+bb$
		$a+b$	$a-b$
4 × a	5	$a^3+2aab+abb$	$a^3-2aab+abb$
4 × b	6	$+aab+2abb+b^3$	$-aab+a2bb-b^3$
1 ⊙ 3	7	$a^3+3aab+3abb+b^3$	$a^3-3aab+3abb-b^3$

Thus are the Square and Cube produced from their Roots; and by this Method of multiplying by the Root, may any Power be raiſed from any Compound Quantities given: Other Methods there are, but this I take to be the plaineſt (and moſt natural) for a Learner's Practice.

Again, Suppoſe the Square of $a+b+c$, was required.

Trinomial Root.

	1	$a+b+c$
		$a+b+c$
1 × a	2	$aa+ab+ac$
1 × b	3	$+ab+bb+bc$
1 × c	4	$+ac+bc+cc$
1 ⊙ 2	5	$aa+2ab+bb+2ac+2bc+cc$

The Uſe, and eaſier Conſtruction of Powers ariſing from 2, 3, 4, or more Members in the Root, ſhall be ſhewed hereafter.

§. 6. *Evolution* of Compound Quantities, is the Converſe of *Involution*, (and conſequently perform'd by contrary Operations.) Whoever intends to be very dextrous at *Evolution*, or extracting of Roots out of Compound Quantities, remaining in their Species, muſt be very ready, and underſtand well the Geneſis, or raiſing of Powers from their Roots; otherwiſe it will be difficult to

difdiſcover whether the given Quantities have a true Root or not.

The Problems in this Tract require no ſuch Operations; yet that it may not appear deficient, obſerve this eaſy Rule.

Take ſo many diſtinct Quantities, as there are concerned in the given Power to be evolved, and involve theſe Quantities (ſo taken) to the height of the given Power, (without reſpect to the Signs;) this new raiſed Power being compared with the given Power, will ſhew whether it have a true Root or not; which if it have, then it's eaſily evolved: If not, then it is a Surd Compound, and muſt have its Sign prefixed to it, and is no otherwiſe to be expreſſed, till it come to be evolved in Numbers.

But becauſe ſuch Operations are tedious, I ſhall omit giving Examples thereof in Species, and according to the Deſign of this Tract, (which is brevity) proceed to Fractions.

CHAP. IV.

Of Fractions, *or Broken Quantities.*

§. 1. *What is done about Vulgar* Fractions *in Numbers, the ſame may alſo be performed in Quantities.*

Rule 1. TO bring whole Quantities to a Fraction of a given Denominator, multiply both together for a Numerator, and place the given Quantity for the Denominator; thus $a = \frac{ad}{d}$, and $b = \frac{bcd}{cd}$, alſo $a = \frac{ac+da}{c+d}$, $d + \frac{dd}{b} = \frac{db+dd}{b}$ &c.

Rule 2. When whole Quantities are to be expreſſed Fraction-wiſe, give them an Unity for a Denominator, as $aa = \frac{aa}{1}$ and $ad+bc = \frac{ad+bc}{1}$.

§. 2. *To reduce* Fractions *to a lower Denomination.*

Rule 1. When the ſame Quantities are found both in the Numerator and Denominator, caſt them away in both, and a Fraction remains of the ſame value; as $\frac{abc}{dbc}=\frac{a}{d}$, and $\frac{bcdf}{bdf}=c$. $x+\frac{adf}{df}=x+a$.

Thus the greateſt common Diviſor brings the Fraction to the loweſt Denomination.

Rule 2. But when the Fraction is a Compound, the Method to obtain that common Diviſor, is by dividing the Denominator by the Numerator, and the Numerator by the Reſidue, and ſo on (as in Vulgar Arithmetick) till there remain nothing: By the laſt Diviſor, divide both the Numerator and Denominator of the Fraction, and it will be reduced to the loweſt Denomination.

Example. Let the Fraction propoſed be $\frac{ab+cb}{aa+2ac+cc}$

	1	$aa+2ac+cc$, the Denominator.
	2	$ab+bc$, the Numerator.
1 ÷ 2	3	$\frac{a+c}{b}$
3 × b	4	$a+c$ { *Note*, any Fraction is multiplied by ſo much as is caſt away out of the Denominator.
		Hence it appears $a+c$ is the greateſt Diviſor,
2 ÷ 4 1 ÷ 4	5 6	$\frac{b}{a+c}$ the new Fraction $=\frac{ab+cb}{aa+2ac+cc}$

But if this way be thought too tedious, then diſcover all the Aliquot Parts in each, and the Denominator and Numerator being divided thereby, are reduced.

But if after all means uſed, there cannot be found one common Diviſor to both, then are the Fractions Prime to each other.

§. 3. *To bring different* Fractions *into one Denomination, retaining the ſame Value.*

Rule 1. THis is performed (as in *Vulgar Fractions*) by multiplying the Denominators into each other for a new Denominator, and the Numerators croſs-wiſe into each other's Denominator, for new Numerators.

Let the Fractions be $\frac{a}{b}$, $\frac{c}{d}$, $\frac{f}{h}$, &c.

Reduced will ſtand thus, $\frac{adh}{bdh}$, $\frac{bch}{bdh}$, $\frac{bdf}{bdh}$, and retain the ſame Value with the firſt.

By theſe means Fractions are made fit for *Addition* and *Subſtraction*; which two Rules cannot well be perform'd, until different Fractions are reduced to one Denomination.

§. 4. Addition of Fractions.

Rule. WHen the Fractions are brought to one Denomination, (as before) add, or ſubſtract their Numerators.

Example.

Thus,	1	$\frac{3b}{d}$	$\frac{3c+2b}{d+f}$	$\frac{a-b+de}{c+f}$
	2	$\frac{a}{d}$	$\frac{x+h}{d+f}$	$\frac{a+b-de}{c+f}$
1+2	3	$\frac{a+3b}{d}$	$\frac{3c+2b+x+h}{d+f}$	$\frac{2a}{c+f}$

Subſtraction.

Thus,	1	$\frac{3b}{d}$	$\frac{3c+2b}{d+f}$	$\frac{a-b+de}{c+f}$
	2	$\frac{a}{d}$	$\frac{x-h}{d+f}$	$\frac{a+b-de}{c+f}$
1−2	3	$\frac{3b-a}{d}$	$\frac{3c+2b-x+b}{d+b}$	$\frac{2de-2b}{c+f}$

§. 5.

§. 5. Multiplication *of* Fractions.

Rule 1. MUltiply the Numerators and Denominators reſpectively into each other, (as in Vulgar Fractions) their Products are the new Fractions.

Note, But firſt prepare them for the Work, by making mix'd Quantities improper Fractions, having firſt brought down the Fractions to their loweſt Terms; alſo make whole Quantities like Fractions, by placing an Unity under them: *As in the firſt Section of this Chapter.*

	1	$\frac{2a}{b}$	$\frac{2a+b}{d-e}$	$\frac{a-7+b}{d}$
	2	$\frac{3c}{d}$	$\frac{de}{d+e}$	$\frac{a+8}{f-g}$
1 × 2	3	$\frac{6ac}{db}$	$\frac{2ade+dbe}{dd-ee}$	$\frac{aa+a+ba+8b-56}{df+dg}$

Whole Quantities by Fractions ſtand as followeth:

Thus,	1	$\frac{b+c}{1}$	$\frac{ab-ad}{1}$	$\frac{ac+7}{1}$
	2	$\frac{cd}{f}$	$\frac{b}{c}$	$\frac{a+c}{b+c}$
1 × 2	3	$\frac{bcd+ccd}{f}$	$\frac{abb-abd}{c}$	$\frac{aac+7a+acc+7c}{b+c}$

§. 6. Diviſion *of* Fractions.

Rule 1. D*iviſion* is thus perform'd: Multiply the Numerator of the Dividend by the Denominator of the dividing Fraction, for a new Numerator, and the other Numerator and Denominator, for a new Denominator.

Suppoſe $\frac{a+b}{d}$ were to be divided by $\frac{c-b}{f}$, it will ſtand thus $\frac{a+b}{d} \div \frac{c-b}{f} = \frac{af+bf}{cd-bd}$.

Again, Admit $aa+bc$ were to be divided by $\frac{cb-ad}{c+g}$, it will be $\frac{aa+bc}{1} \div \frac{cb-ad}{c+g} = \frac{aac+bcc+aag+bcg}{cb-ad}$.

Rule 2. When the Fractions are of one Denomination, divide the Numerators, and caſt off the Denominator; as $\frac{aacc}{b} \div \frac{ac}{b} = ac$, for $\frac{aaccb}{acb} = ac$.

Again let $\frac{abb}{d} \div \frac{abc}{c} = \frac{b}{d}$ for $\frac{abbc}{abcd} = \frac{b}{d}$.

§. 7. Involution *of* Fractions, *is performed in all reſpects like* Multiplication.

Thus,	1	$\frac{3a}{b}$	$\frac{4bd}{c}$	$\frac{a+b}{g+x}$
1 ⊙ 2	2	$\frac{9aa}{bb}$	$\frac{16bbdd}{cc}$	$\frac{aa+2ab+bb}{gg+2gx+xx}$
1 ⊙ 3	3	$\frac{27aaa}{bbb}$	$\frac{64bbbddd}{ccc}$	$\frac{a^3+3baa+3bba+b^3}{g^3+3ggx+3gxx+x^3}$

N. B. If after any of theſe Operations in the foregoing Parts of *Addition, Subſtraction*, &c. there be like Quantities in the Numerator and Denominator, care is to be taken that ſuch Fractions be reduced lower, by ſome of the former Rules, if poſſible, before they be engaged in other Operations.

I ſhall paſs over *Evolution*, becauſe it rarely falls out, that both the Numerator and Denominator have a true Root; therefore if the Sign of the Root be prefixed, it's ſufficient, until ſuch Fractions are evolved in Numbers.

CHAP. V.

Of Surd Quantities.

I Don't pretend to (nor can it be expected that I ſhould) give a full account of *Surd Quantities* (*with all their tedious Reductions*) in this Compendium: Nor is it at all needful, ſince the Method of Reſolving all ſorts of Equations, is here render'd equally eaſy in all Caſes.

However, I think it convenient to ſhew the young Algebraiſt, how to manage ſuch *Surd Quantities* as may, and often do riſe in the Solution of hard difficult Queſtions.

§. 1. *Addition of Surd Quantities.*

Caſe 1. When the *Surd Quantities* are Homogeneal; that is, are alike in Power; add the Rational Part prefix'd to the *Surd*, if there be any; and to that Sum, adjoin the Irrational or *Surd Quantity*.

Examples.

	1	$6\sqrt{ab}$	$-4b\sqrt{cd}$	$d\sqrt{bb+cc}$
	2	$8\sqrt{ab}$	$-8b\sqrt{cd}$	$5d\sqrt{bb+cc}$
1 + 2	3	$14\sqrt{ab}$	$-12b\sqrt{cd}$	$6d\sqrt{bb+cc}$

	1	$3d\sqrt[3]{aab}$	$f+\sqrt[5]{d+cc}$
	2	$4d\sqrt[3]{aab}$	$d-\sqrt[5]{d+cc}$
1 ÷ 2	3	$7d\sqrt[3]{aab}$	$f+d$

Concerning Subſtraction of Surds, &c.

If the *Surds* are Homogeneal, as before, ſubſtract the Rational Part, and to the Remainder join the *Surd*, as in *Addition*.

Examples in Subſtraction.

	1	$14\sqrt{ab}$	$-12b\sqrt{cd}$	$6d\sqrt{bb+cc}$
	2	$8\sqrt{ab}$	$-4b\sqrt{cd}$	$5d\sqrt{bb+cc}$
$1-2$	3	$6\sqrt{ab}$	$-8b\sqrt{cd}$	$d\sqrt{bb+cc}$

	1	$7d\sqrt[3]{aab}$	$f+d$	$2a\sqrt{b}$
	2	$4d\sqrt[3]{aab}$	$d-\sqrt[5]{d+cc}$	$-a\sqrt{b}$
$1-2$	3	$3d\sqrt[3]{aab}$	$f+\sqrt[5]{d+cc}$	$+3a\sqrt{b}$

Caſe 2. When the *Surd Quantities* are Heterogeneal, that is, when their Indices are unlike, they are only to be added, or ſubſtracted by their Signs of +, or —. And their Sum, or Difference, will produce either Binomial, or Reſidual Surds; as in theſe

Examples in Addition.

	1	$\sqrt{ba}$	$5dd\sqrt[3]{ab+dd}$
	2	$\sqrt{da}$	$3dd\sqrt{ac+ab}$
$1+2$	3	$\sqrt{ba}+\sqrt{da}$	$5dd\sqrt[3]{ab+dd}+3dd\sqrt{ac+ab}$

Examples in Subſtraction.

	1	$\sqrt{ba}$	$d-b\sqrt{ab}$
	2	$\sqrt{da}$	$c-f\sqrt{dca}$
$1-2$	3	$\sqrt{ba}-\sqrt{da}$	$d-b\sqrt{ab}-c+f\sqrt{dca}$

§. 2. *Multiplication of Surd Quantities,*

Caſe 1. When the Quantities are only *Surds* of the ſame kind; multiply them together: and to their Product prefix the common Radical Sign.

Examples.

	1	$\sqrt{a}$	$\sqrt{dc+fa}$	$\sqrt{ma+b}$
	2	$\sqrt{d}$	$\sqrt{ba}$	$\sqrt{ma-b}$
1×2	3	$\sqrt{ad}$	$\sqrt{dcba+bfaa}$	$\sqrt{mmaa-bb}$

Caſe 2. When *Surd Quantities* of the ſame kind (*as before*) are join'd to Rational Numbers or Quantities, multiply the Rational into the Rational, and the *Surds* into the *Surds* (as above) then join their Products together.

Examples.

	1	$2a\sqrt{bc}$	$5cb\sqrt{fa-d}$
	2	$3d\sqrt{ab}$	$3ad\sqrt{fa+d}$
1 × 2	3	$6ad\sqrt{bbac}$	$15abcd\sqrt{ffaa-dd}$

§. 3. Diviſion *of Surd Quantities.*

Caſe 1. When *Surd Quantities* of the ſame kind can be divided without leaving any Remainder, divide them, and to their Quotient prefix their Radical Sign; *viz.* the Index of their Power.

Examples.

	1	$\sqrt{ad}$	$\sqrt{dcba+bfaa}$	$\sqrt{aaa}$
	2	$\sqrt{a}$	$\sqrt{dc+fa}$	$\sqrt{a}$
1 ÷ 2	3	$\sqrt{a}$	$\sqrt{ba}$	$\sqrt{aa}=a$

Caſe 2. When the *Surd Quantities*, of the ſame kind, are join'd to Rational Quantities, divide the Rational by the Rational, if that can be done; and to their Quotient, join the Quotient of the *Surd*, divided by the *Surd*, with its firſt Radical Sign.

Examples.

	1	$6ad\sqrt{bbac}$	$15abcd\sqrt{ffaa-dd}$
	2	$3d\sqrt{ab}$	$3ad\sqrt{fa+d}$
1 ÷ 2	3	$2a\sqrt{bc}$	$5bc\sqrt{fa-d}$

In Diviſion it muſt needs be eaſy to conceive, that if any Square be divided by its Root, the Quotient will be the Root again.

Examples.

	1	ab	$bb - 2bc + cc$	$25aabb$
	2	$\sqrt{ab}$	$\sqrt{bb - 2bc + cc}$	$\sqrt{25aabb} = 5ab$
$1 \div 2$	3	$\sqrt{ab}$	$\sqrt{bb - 2bc + cc}$	$\sqrt{25aabb} = 5ab$

§ 4. Involution *of Surd Quantities.*

Caſe 1. If the *Surd Quantities* are not join'd to Rational Quantities, they are involved to the ſame height as their Index denotes, by only taking away that Index.

Examples.

	1	$\sqrt{a}$	$\sqrt{cda}$	$\sqrt{5b + db}$	$\sqrt{bb - aa}$
$1 \circledcirc 2$	2	a	cda	$5b + db$	$bb - aa$

Caſe 2. When the *Surd Quantities* are join'd to Rational Quantities, involve the Rational to the ſame Power as the Index of the *Surd* denotes; then multiply thoſe involved Quantities into the *Surd Quantities*, after their Radical Sign is taken away, as before.

Examples.

	1	$d\sqrt{a}$	$6b\sqrt{ca}$	$7d\sqrt{aa + bb}$
$1 \circledcirc 2$	2	dda	$36bbca$	$49ddaa + 49ddbb$
	1	$a\sqrt[3]{d}$	$da\sqrt[3]{b}$	$4c\sqrt[3]{aa - bb}$
$1 \circledcirc 3$	2		$dddaaab$	$64cccaa - 64cccbb$

The Reaſon why only taking away the Radical Sign involves the *Surd*, (as in *Caſe* 1.) will be eaſy to conceive, when you conſider, that any Root involved into itſelf, produces a Square, and ſo on to higher Powers.

And the Reaſon of the Work, in *Caſe* 2. may be thus ſtated: Suppoſe $d\sqrt{a} = f$, then $\sqrt{a} = \frac{f}{d}$. *per Axiom* 4. and if both ſides of the Equation be equally involved, it will become $a = \frac{ff}{dd}$. Then multiplying both ſides into dd, it will $dda = ff$.

CHAP. VI.

Of Proportions.

§. 1. PRoportion is either Disjunct, or Continued; Disjunct is, when of four Quantities, the firſt is to the ſecond, as the third is to the fourth; if Quantities be thus qualified, then the Product of the two Extremes is equal to the Product of the two Means. And if the Product of the Extremes be equal to the Product of the Means, then are thoſe Quantities proportional, (16. *Eucl.* 6.) let the four Quantities propoſed, be a, b, c, d, if theſe be proportional, they ſhall alſo be proportional, being *Alterned*, *Inverted*, *Compounded*, *Divided*, *Converted*, and in *Mixture*.

1 $a : b :: c : d$, Direct.
1, ∵ 2 $ad = bc$

3 $a : c :: b : d$, Alterned.
3, ∵ 4 $ad = bc$

5 $b : a :: d : c$, Inverted.
5, ∵ 6 $bc = ad$

7 $a + b : b :: c + d : d$, Compounded.
7, ∵ 8 $ad + bd = bc + bd$, that is, $ad = bc$.
Or 9 $a + c : c :: b + d : d$.
9, ∵ 10 $ad + cd = bc + cd$, that is $ad = bc$.

11 $a - b : b :: c - d : d$, Divided.
11, ∵ 12 $ad - bd = bc - bd$, that is, $ad = bc$.
Or 13 $a - c : c :: b - d : d$.
13, ∵ 14 $ad - cd = cb - cd$, that is, $ad = bc$.

15 $a : b \pm a :: c : d \pm c$, Converted.
15, ∵ 16 $ad \pm ac = bc \pm ac$, that is, $ad = bc$.

17 $a + b : a - b :: c + d : c - d$, in Mixture.
17, ∵ 18 $ac - ad + bc - bd = ac + ad - bc - bd$.
18, ± 19 $2bc = 2ad$, that is, $ad = bc$.

From the foregoing Operations, it is evident, that four proportional Quantities may be varied, and intermingled one with the other, and yet retain their true Proportions; which variation may be of good uſe upon ſeveral Occaſions.

N.B. And from hence may be learnt, how to convert Analogies into Equations, or the contrary of converting Equations into Analogies.

§. 2. Continued Proportion (or Geometrical Progreſſion) is, when of Three, Four, Five, or more Quantities, the Firſt is to the Second, as that Second is to the Third; and the Second is to the Third, as that Third is to the Fourth, &c. that is, when all the middle Terms between the Firſt and Laſt Term are both Conſequents and Antecedents; ſuch are in ∺.

as, $a : b :: b : \frac{bb}{a} :: \frac{bb}{a} : \frac{bbb}{aa} :: \frac{bbb}{aa} : \frac{bbbb}{aaa} :: \frac{b^5}{aaaa}$ &c.

Theſe Quantities are in continued Proportion: For the Product of any two of the Means, is equal to the Product of the two Extremes that are equally diſtant from thoſe Means.

viz. $a \times \frac{bb}{a} = b \times b$. And $b \times \frac{bbb}{aa} = \frac{bb}{a} \times \frac{bb}{a}$, &c.

Therefore they are Proportionals. (19. *E.* 7.)

And here, $a + b + \frac{bb}{a} + \frac{bbb}{aa} + \frac{bbbb}{aaa}$ is the Sum of all the Antecedents.

And, $b + \frac{bb}{a} + \frac{bbb}{aa} + \frac{bbbb}{aaa} + \frac{bbbbb}{aaaa}$ the Sum of all the Conſequents.

But if never ſo many Quantities are in continued Proportion; it will always be, As one of the Antecedents: Is to its Conſequent :: So is the Sum of all the Antecedents: To the Sum of all the Conſequents.

That is, $a : b :: a + b + \frac{bb}{a} + \frac{bbb}{aa} + \frac{bbbb}{aaa}$:

To $b + \frac{bb}{a} + \frac{bbb}{aa} + \frac{bbbb}{aaa} + \frac{bbbbb}{aaaa}$, &c.

CHAP.

CHAP. VII.

Of Equations, *and Queſtions reſolved thereby.*

§. 1. QUantities are ſaid to be brought to an Equation, when ſome one of thoſe that be ſought, (or ſome Powers thereof, as Squares, Cubes, &c.) is found equal to thoſe that be given. And this is to be obtained by a mutual comparing of equal Quantities (or things) with equal, by ſuch Methods or Rules, as the Nature of the Problem (or Queſtion) requireth.

Therefore, when any Queſtion is propoſed to be reſolved, it is highly requiſite that the Deſign or Meaning thereof, be fully and clearly comprehended, that ſo the Queſtion may be truly ſtated, or fitted for Operation.

And this part of the Work is ſomething difficult; yet having a competent Knowledge in the Principles of *Geometry* and *Arithmetick* (together with a good and deſiring Genius) a little Practice will render it facile and pleaſant.

Having got a clear underſtanding of the Queſtion propoſed, for each Quantity concerned therein, be it a Line, or Number, put or ſubſtitute a Letter, as before taught at the beginning of this Tract, (*page* 12, & 13.) But if two, or more Quantities in the Queſtion are granted to be equal, they may be expreſſed by the ſame Letter.

Having placed down all the concerned Quantities in their due order (according to the Import of the Queſtion) then, conſider whether the ſame be limited or not; for the diſcovery thereof, obſerve the following Rules.

Rule 1. Whenſoever the Number of required Quantities exceeds the Number of the given Equations, the Queſtion is capable of innumerable Anſwers.

For example; Suppoſe a Queſtion were thus propoſed: There be Three Numbers, the Firſt is equal to the Square of the Second, and the Sum of the Firſt and Second is equal to the Third; What are thoſe Three Numbers?

To ſignify theſe Three Numbers, put three Letters; call the Firſt a, the Second e, and Third y; then $a=ee$, and $a+e=y$, according to the Queſtion. Here the ſought Quantities are Three, to wit, a, e, and y, but the given Equations are but Two; therefore this Queſtion is unlimited: becauſe for any one of theſe Three Quantities, any Number may be taken at pleaſure. Therefore, unlimited.

Rule 2. But when the given Equations (not depending one upon the other) are as many as the required Quantities, then hath the Queſtion a determinable number of Anſwers; for then each Quantity concern'd, hath but one ſingle Value.

And if the given Equations be more than the ſought Quantities, they not only limit the Queſtion, but oftentimes render it unreſolvable, by being inconſiſtent one with another.

The Queſtion being truly ſtated, and found limited to one Anſwer, (or at leaſt ſo bounded, as to have a certain determinable number of Anſwers) let all the Quantities be ſo ordered, moulded or compared, by adding, ſubducting, multiplying, or dividing them, until there remain but one unknown Quantity, (or ſome Powers thereof) equal with the given Quantities. And all the reſt of the unknown Quantities concern'd in the Queſtion, are caſt off or vaniſhed; which indeed is the chiefeſt buſineſs required in the forming of an Equation, and therefore ought to be well conſidered: but no Rules can be preſcribed to ſuit all Caſes.

And next, becauſe that in moſt (if not all) Equations, when firſt they appear out of the Covers of the Queſtion, the known and unknown Quantities are intangled and mixt together, on both ſides of the Equation; it is thereſore requiſite ſo to order and clear ſuch Equations, that one ſide thereof may conſiſt of known Quantities only, and the other ſide of the unknown Quantities, (ſtill keeping them to a juſt equality) which Work fits the Queſtion for a Solution in Numbers, and is not improperly called *Reduction*. Which will be eaſily perform'd, if the following *Axioms* be once clearly underſtood.

AXIOMS.

1. If Equal Quantities be *added* to Equal Quantities; the *Sum* of thoſe Quantities will be Equal.

2. If Equal Quantities be *taken* from Equal Quantities, the remaining Quantities will be Equal.

3. If Equal Quantities be multiplied with Equal Quantities; their Products will be Equal.

4. If Equal Quantities be divided by Equal Quantities; the Quotient Quantities will be Equal.

5. Thoſe Quantities, that are Equal to one and the ſame Thing, are Equal to one another.

Upon the true Knowledge of theſe Five *Axioms*, depends, not only the Work of this Chapter, but all that follows it. And therefore I adviſe my young Algebraiſt to underſtand them well, before he proceed further.

By Addition.

Rule 1. WHEN any Quantity is connected with the Sign —, it is added by caſting away the Sign —, and tranſferring the Quantity to the other ſide of the Equation, with the Sign +.

Thus,	1	$a - b - 15 = 60$	*per Axiom* 1.
$1 + 15$	2	$a - b = 75$	
$2 + b$	3	$a = 75 + b$	

Again,	1	$aa - bc - d = cc - ba$
$1 + ba$	2	$aa + ba - bc - d = cc$
$2 + bc$	3	$aa + ba - d = cc + bc$
$3 + d$	4	$aa + ba = cc + bc + d$

By Subſtraction.

Rule 2. When any Quantity is connected with the Sign +, it is to be ſubſtracted, by caſting away the Sign +, and tranſferring the Quantity to the other ſide of the Equation, with the Sign —

Thus,	1	$a + b + c = d$	*per Axiom* 2.
$2 - c$	2	$a + b = d - c$	
$1 - b$	3	$a = d - c - b$	

Theſe two Rules are called Tranſpoſition of Quantities.

By Multiplication.

Rule 3. By this Rule, part (or the whole) being in Fractions, are brought into whole Quantities.

Thus,	1	$a+b=\frac{dd}{a-b}$	*per Axiom* 3.
$1 \times a-b$	2	$aa-bb=dd$	
$2+bb$	3	$aa=dd+bb$	

By Division.

Rule 4. When there is any Quantity known or unknown concerned in every Member of an Equation, divide the whole Equation by ſuch Quantity, and it will clear one Member thereof (at leaſt) and bring down the reſt.

Thus,	1	$aab+bca=bd$	*per Axiom* 4.
$1 \div b$	2	$aa+ca=d$	

Again,	1	$aab+ca=da$
$1 \div a$	2	$ab+c=d$
$2-c$	3	$ab=d-c$
$3 \div b$	4	$a=\frac{d-c}{b}$

Again,	1	$baaa+bbaa-ca=da$
$1+c$	2	$baa+bba-c=d$
$2+c$	3	$baa+bba=d+c$
$3 \div b$	4	$aa+ba=\frac{d+c}{b}$

By Involution.

Rule 5. When there is an Equation between like Surds, (or Radical Signs) caſt away the Radical Signs, and the Quantities are thereby reduced to Rational ones.

Thus,	1	$aa\sqrt{b}=b\sqrt{da}$
1 ⊙ 2	2	$aaaab=bbda$
$2 \div ab$	3	$aaa=bd$

Again,	1	$b\sqrt[3]{aa}=\frac{dc}{b}$
1 ⊚ 3	2	$bbbaa=\frac{dddccc}{bbb}$
2 ÷ b	3	$aa=\frac{dddccc}{bbbbbb}$

Alſo,	1	$\sqrt[2]{\frac{baa+3ba-d}{4}}=\frac{b+d}{2}$
1 ⊚ 2	2	$\frac{baa+3ba-d}{4}=\frac{bb+2bd+dd}{4}$
2 × $\overline{4}$	3	$baa+3ba-d=bb+2bd+dd$
3 + d	4	$baa+3ba=bb+2bd+dd+d$
4 ÷ b	5	$aa+3ab=\frac{bb+2bd+dd+d}{b}$

By Evolution.

Rule 6. By this Rule, Equations are oft brought down to lower Powers, thus:

	1	$aa+2ba+bb=d$
1 ɯ 2	2	$a+b=d$

But if the Equation had been $aa+2ba=d$, which is an imperfect Square, and wants the Square of bb to fill it up, before it can be brought down as above; ſuch Operation Dr. *Pell* calls, *Compleating the Square*: and when it is performed, uſeth this Sign *c*□ put in the Margin.

As,	1	$aa+2ba=d$
1, *c*□	2	$aa+2ba+bb=d+bb$
2 ɯ 2	3	$a+b=\sqrt{d+bb}$
3 − b	4	$a=\sqrt{d+bb}-b$

Suppoſe,	1	$aa-ba=d$
1, *c*□	2	$aa-ba+\frac{1}{4}bb=d+\frac{1}{4}bb$
2 ɯ 2	3	$a-{}_{2}ba=\sqrt{d+\frac{1}{4}bb}$
3 + ½b	4	$a=\sqrt{d+\frac{1}{4}bb}+\frac{1}{2}b$

In theſe it is evident, that the laſt place of bb, or that of $\frac{1}{4}bb$, is produced, by involving half the Coefficient b, of the middle place.

§. When the Equations are thus cleared or reduced, then are they fit ro be reſolved into Numbers, and are either Pure and Simple, or Mixt and Adfected. Pure and Simple Equations are ſuch as have the unknown Quantity (a) at one ſide thereof:

As $a = b + c$, or $a + b = d$, &c.

Alſo when there is only ſome Power of (a)

As $aa = b$, and $aaa = c$, or $aaaa = d$, &c.

Mix'd and Adfected, are ſuch as have the unknown Quantities at one ſide of the Equation, in different Degrees or Powers thereof;

As $aa + ba = d$, and $aa - ba = d$; or $ba - aa = d$.

Theſe are called the three Forms of Square Adfected Equations.

Again, $aaa + ba = c$, and $aaa - ba = c$, or $ba - aaa = c$.

Theſe are called the three Forms of Cubick Adfected Equations.

Infinite other Varieties there are, ſuch

As $aaa + baa + ca = d$, and $aaa - baa = d$.
Or $aaaa + baaa + caa - da = f$, &c.

How theſe, or any other, of all Degrees, or howſoever Mixt and Adfected, may be eaſily reſolved into Numbers, ſhall be effectually ſhewed further on, when we come to the Reſolving of ſuch Problems as will produce the ſame; taking each in their Order. *viz.* Firſt, I will ſhew how to Reſolve (or extract the Roots of) all pure Equations; next, the three Forms of Square Adfected Equations; the like in Adfected Cubicks; and ſo on to others of different degrees.

But before we proceed to that part, it will be convenient to inſert a few eaſy Queſtions in Numbers, thereby to explain, and (in part) ſhew the Uſe of the foregoing Rules. The which will be as an Introduction to the ſucceeding Problems.

CHAP. VIII.

Containing ſeveral eaſy Queſtions in Numbers, fit for a Learner's Practice.

Queſt. 1. THERE be two unknown, and unequal Numbers, whoſe Sum is $53 = s$, and their Difference is $21 = d$; What are thoſe Numbers?

For the greater put a, and for the Leſſer put e.

Then according to the Queſtion, they will ſtand in their Subſtitutes thus:

		Quantities.	Numbers.
	1	$a + e = s$	$= 53$
	2	$a - e = d$	$= 21$
$1 + 2$	3	$2a = s + d$	$= 74$
$3 \div \overline{2}$	4	$a = \frac{s + d}{2}$	$= \frac{74}{2} = 37$
$1 - 2$	5	$2e = s - d$	$= 32$
$5 \div \overline{2}$	6	$e = \frac{s - d}{2}$	$= 16$

Hence it appears, that 37 and 16 are the two Numbers ſought.

Note, When any Number is placed in the Margin, with a Line over it, as $\overline{2}$ in the fourth and ſixth Steps, ſuch is an abſolute Number, and hath no relation to thoſe Numbers in the ſmall Column.

Here you may perceive, that the firſt Work is to caſt off one of the unknown Quantities; and becauſe the Quantity e hath both Signs, + and — prefix'd to it, therefore by the Rule of *Addition* it may be vaniſhed, as appears by the Work of the third Step.

Queſt. 2. There be two Numbers (as before) whoſe Sum is $56 = s$, and the Greater (a) divided by the Leſſer (e) the Quotient is $2{,}5 = q$; What are the Numbers? a, and e.

	1	$a+e=s$	by the Queſtion.
	2	$a\div e$, or $\frac{a}{e}=q$	
$1-a$	3	$e=s-a$	
$2\times e$	4	$qe=a$	
$4\div q$	5	$e=\frac{a}{q}$	
3, 5,	6	$s-a=\frac{a}{q}$	
$2\times q$	7	$qs-qa=a$	
$7+qa$	8	$qa+a=qs$	
$8\div\overline{q+1}$	9	$a=\frac{qs}{q+1}$ for $q+1\times a=qa+1a$	
$1-9$	10	$e=s-\frac{qs}{q+1}=\frac{s}{q+1}$.	

For an Explanation, take the ſame in Numbers.

	1	$a+e=56$
	2	$a\div e$, or $\frac{a}{e}=2{,}5$
$1-a$	3	$e=56-a$
$2\times e$	4	$2{,}5\,e=a$
$4\div\overline{2{,}5}$	5	$e=\frac{a}{2{,}5}=56-a$
$5+a$	6	$a+\frac{a}{2{,}5}=56$
$5\times\overline{2{,}5}$	7	$2{,}5a+a=140$
$7\div\overline{3{,}5}$	8	$a=\frac{140}{3{,}5}=40$ That is, by $2{,}5+1$
3,	9	$e=56-40=16$

Queſt. 3. Let there be given the Difference of the aforeſaid two Numbers, to wit, $24=d$, and the Quotient, $2{,}5=q$, from thence to find out the two Numbers.

	1	$a - e = d$ } by the Queſtion.
	2	$\frac{a}{e} = q$
$1 + e$	3	$a = d + e$
$2 \times e$	4	$a = qe$
3, 4,	5	$qe = d + e$
$5 - e$	6	$qe - e = d$
$6 \div q - 1$	7	$e = \frac{d}{q - 1}$
$1 + 7$	8	$a = d + \frac{d}{q - 1} = \frac{qd}{q - 1}$

The reducing of this into Numbers, is eaſy, (the foregoing Queſtions conſidered.)

Queſt. 4. There be two Numbers, their Sum is $56 = s$, and the Difference of their Squares is $1344 = x$; What are thoſe Numbers?

	1	$a + e = s$ } by the Queſtion.
	2	$aa - ee = x$
$2 \div 1$	3	$a - e = \frac{x}{s}$ for $a + e \times a - e = aa - ee$
$1 + 3$	4	$2a = s + \frac{x}{s} = \frac{ss + x}{s}$
$4 \div 2$	5	$a = \frac{ss + x}{2s}$
$1 - 3$	6	$2e = s - \frac{x}{s} = \frac{ss - x}{s}$
$6 \div 2$	7	$e = \frac{ss - x}{2s}$

Hence (a) is found equal to (40) and (e) equal to (16) by the 5th and 7th Steps.

Queſt. 5. Two Men have each a Sum of Money, the one hath four times as much as the other; both their Sums put together, will not make 100 *l.* but if they be doubled, and from that double be taken 30 *l.* the Remainder will be twice as much above 100 *l.* as before they wanted of 100 *l.* How much hath each Man?

Put *a* for the Greater, and *e* for the Leſſer.

Then	1	$a = 4e$	*d* ſignifieth the difference their Sums wanted of 100 *l.*
	2	$a + e = 100 - d$	
And	3	$2a + 2e - 30 = 100 + 2d$, by the Queſtion.	
2×2	4	$2a + 2e = 200 - 2d$	
$3 + 4$	5	$4a + 4e - 30 = 300$	
$5 + 30$	6	$4a + 4e = 330$	
1, 6,	7	$5a = 330$	
$7 \div 5$	8	$a = \frac{330}{5} = 66.$	
1, 8,	9	$4e = 66$	
$9 \div 4$	10	$e = \frac{66}{4} = 16,5$	

Hence it is found, that one Man had 66 *l.* and the other 16 *l.* 10 *s.* which two Sums will anſwer the Queſtion in each particular.

Note, That as *e*, in the firſt Queſtion, was caſt off by *Addition*, (for the Reaſons there mentioned) ſo in the ſecond, the like was done by *Multiplication* and *Diviſion*; in the third, by *Multiplication* and *Addition*; in the fourth, by *Diviſion* and *Addition*; and in this Queſtion by *Multiplication* and *Equality*.

Now if the Reaſon of theſe be conſidered, and once underſtood, the whole buſineſs of forming any Equation will be eaſy; for the only Myſtery therein, lies in caſting off all the unknown Quantities but one, (of which, ſee *Pag.* 39.)

Queſt. 6. Three Men part their Money in this manner: The Firſt gives to the Second and Third, as much as they have about them; the Second gives to the Firſt and Third as much as now they have; and the Third doth the like to the Firſt and Second: after they had done thus, each Man had 20 Shillings. The Queſtion is, what each Man had at firſt?

For the Firſt Man's Sum put *a*, for the Second *e*, and for the Third *y*.

Then

Then	1	$a - e - y$	is what the Firſt hath left after his Gift.
	2	$2e$	is what the Second and Third have, after the Firſt Man's Gift.
	3	$2y$	
$2 - 1 - 3$	4	$3e - a - y$	
$1 \times \overline{2}$	5	$2a - 2e - 2y$	the Firſt and Third doubled by the Second.
$3 \times \overline{2}$	6	$4y$	
$6 - 4 - 5$	7	$7y - a - e$	and ſo much each hath at laſt, *viz.* = 20.
$5 \times \overline{2}$	8	$4a - 4e - 4y$	
$4 \times \overline{2}$	9	$6e - 2a - 2y$	
That is	10	$7y - a - e = 20$	
	11	$4a - 4e - 4y = 20$	
	12	$6e - 2a - 2y = 20$	
$11 \div \overline{4}$	13	$a - e - y = 5$	
$12 \div \overline{2}$	14	$3e - a - y = 10$	
$13 + 14$	15	$2e - 2y = 15$	
$10 + 13$	16	$6y - 2e = 25$	
$15 + 16$	17	$4y = 40$	
$17 \div \overline{4}$	18	$y = 10$	
$15 + 2y$	19	$2e = 15 + 2y = 35$	
$19 \div \overline{2}$	20	$e = \frac{35}{2} = 17{,}5$	
13, 18, 20,	21	$a = 32{,}5$	

Anſw. The Firſt Man had 32 Shillings, 6 Pence; the Second had 17 Shillings, 6 Pence; and the Third had 10 Shillings.

Queſt. 7. There be Three Numbers with theſe Properties, *viz.* If to the Firſt and Second be added half the Third, that Sum will be = *s*, (ſuppoſe 46, or any Number at pleaſure) and if one Third part of the Firſt be added to the Second and Third, that Sum = *s*, as before: alſo if a Fourth part of the Second be added to the Firſt and Third, that Sum will be = *s*, as was the others. What are theſe Numbers?

For thoſe three Numbers, put *a*, *e*, and *y*.

Then

Then	1	$a+e+\frac{1}{2}y=s$	ſtated according to the Queſtion.
	2	$\frac{1}{3}a+e+y=s$	
	3	$a+\frac{1}{4}e+y=s$	
$1\times\overline{2}$	4	$2a+2e+y=2s$	Thus all are brought out of their Fractions.
$2\times\overline{3}$	5	$a+3e+3y=3s$	
3×4	6	$4a+e+4y=4s$	
$4+6$	7	$6a+3e+5y=6s$	
$7-5$	8	$5a+2y=3s$	
$8-5a$	9	$2y=3s-5a$	
$9\div\overline{2}$	10	$y=\frac{3s-5a}{2}$	
$4+5$	11	$3a+5e+4y=5s$	
$11-6$	12	$-a+4e=s$	
$12+a$	13	$4e=s+a$	
$13\div\overline{4}$	14	$e=\frac{s+a}{4}$	
$10+14$	15	$e+y=\frac{3s-5a}{2}+\frac{s+a}{4}=\frac{7s-9a}{4}$	
$15\times\overline{3}$	16	$3e+3y=\frac{21s-27a}{4}$	
5, 16,	17	$a+\frac{21s-27a}{4}=3s$	
$17\times\overline{4}$	18	$4a+21s-27a=12s$	
18 ±	19	$23a=9s$, if (s) be put $=46$, then	
$19\div\overline{23}$	20	$a=\frac{9s}{23}=18$, and $e=16$, and $y=24$.	

Queſt. 8. There be Four Numbers, their Sum is $=s$, (any Number at pleaſure, ſuppoſe 100) and if to the Firſt be added $b=7$, (or any Number) from the Second be ſubducted b, the Third multiplied by b, the Fourth divided by b; it's required that ſuch Sum, Difference, Product and Quotient be equal to each other. What are the Four Numbers?

For the Firſt put a, for the Second e, for the Third y, and for the Fourth u.

Then	1	$a+e+y+u=s$	according to the Propoſals in the Queſtion.
and	2	$a+b=e-b$	
	3	$yb=\frac{u}{b}=a+b$	
2 + b	4	$a+2b=e$	
3 ÷ b	5	$y=\frac{a+b}{b}$	
3,	6	$\frac{u}{b}=a+b$	
6 × b	7	$u=ba+bb$	
1,4,5,6,	8	$a+a+2b+\frac{a+b}{b}+ba+bb=s$	
8,	9	$bba+2ba+a=bs-b^3-2bb-b$	
9 ÷	10	$a=bs-bbb-2bb-b\div bb+2b+1$	
		Suppoſe $s=100$, and $b=7$	
then		$\frac{bs-bbb-2bb-b=252}{bb-2b+1=64}=3{,}9375=a$	
4,	11	$a+2b=17{,}9375=e$	
5,	12	$\frac{a+b}{b}=\frac{10{,}9375}{7}=1{,}5625=y$	
7,	13	$ba+bb=76{,}5625=u$	

Anſw. The Firſt Number is 3,9375; the Second is 17,9375; the Third is 1,5625; and the Fourth is 76,5615.

Queſt. 9. A Man bought a fine Horſe; and ſold him again for Fifty-ſix Pounds. He gain'd as much *per Cent.* by the Bargain, as the Horſe coſt him. *Quære* what he gave for the Horſe, *&c.*

Put	1	$a=$ the Firſt Price.
And	2	$e=$ his Gain by the Sale.
1 + 2	3	$a+e=56$ *l.* by the Queſtion.
3 − a	4	$e=56-a$
But	5	$100 : a :: a : e$ *per* Queſt.
5 ∵	6	$\frac{aa}{100}=e=56-a$
6 ×	7	$aa=5600-100a$
7 + 100a	8	$aa+100a=5600$
8 c □	9	$aa+100a+2500=8100$
9 ω 2	10	$a+50=\sqrt{8100}=90$
10 − 50	11	$a=90-50=40$

viz. The Horſe coſt him 40 *l.* and he gain'd $16=56-40$.

Proof, $100 : 40 :: 40 : 16$

Queſt. 10. Three Joiners undertook to wainſcot a Gentleman's Houſe in 150 Days. One of them (*being eſteem'd the beſt*) was to have 5 *s.* a day for every day he work'd; another was to have 4 *s.* 6 *d.* a day; and the third was to have but 4 *s.* for every day he wrought.

When the Work was finiſh'd, every one of them had juſt the ſame Sum of Money to receive; *Quære*, how many Days each of them work'd? *&c.*

Let *a*, *e*, *y*, repreſent the Days each Man work'd.

Then	1	$a+e+y=150$ by the Queſtion.
And	2	$60a=54e=48y$ by the Queſtion.
$2\div\overline{6}$	3	$10a=9e$
and	4	$10a=8y$
$3\div\overline{9}$	5	$\frac{10a}{9}=e$
$4\div\overline{8}$	6	$\frac{10a}{8}=y$
1, 5, 6	7	$a+\frac{10a}{9}+\frac{10a}{8}=150$
$7\times\overline{9}$	8	$19a+\frac{90a}{8}=1350$
$8\times\overline{8}$	9	$242a=10800$
$9\div 242$	10	$a=\frac{10800}{242}=44,6281$
$10\times\overline{10}$	11	$10a=446,281$
$11\div\overline{9}$	12	$e=49,5868$ from the 5th Step.
$10\div\overline{8}$	13	$y=55,7851$ *per* the 6th Step.

From the 10th Step, *a* wrought $44^{\text{days.}}\ 15^{\circ}\ 4'\ 28''$ and received 11 *l.* 3 *s.* 1,68 *d.*

From the 12th Step, *e* wrought $49^{\text{days.}}\ 14^{\circ}\ 5'\ 00''$ and received 11 *l.* 3 *s.* 1,68 *d.*

By the 13th Step, *y* wrought $55^{\text{days.}}\ 18^{\circ}\ 50'\ 53''$ and received the ſame Sum, as *a*, or *e*.

$$\text{For}\left\{\begin{array}{l}44,6281 \times 60 = 2677,68\\ 49,5868 \times 54 = 2677,68\\ 55,7851 \times 48 = 2677,68\end{array}\right\}\text{Pence} = \text{\&c.}$$

Queſt. 11. There are three Numbers in ∺ Their Sum is 168: which being divided by each of theſe Numbers, the Sum of the three Quotients will be 27$\frac{9}{16}$. What are thoſe Numbers?

Let *a*, *e*, *y* repreſent the three Numbers.

Then	1	$a + e + y = 168 = z$
And	2	$\frac{z}{a} + \frac{z}{e} + \frac{z}{y} = 27\frac{9}{16}$ or $27,5625 = b$
Analo.	3	$a : e :: e : y$
3 ∵	4	$ay = ee$
2 ×	5	$zey + zay + zae = baey$
4 × z	6	$zay = zee$
5, 6	7	$zey + zee + zae = baey$
7 ÷ e	8	$zy + ze + za = bay$
4 × b	9	$bay = bee$
8, 9	10	$zy + ze + za = bee$
10 ÷ z	11	$y + e + a = \frac{bee}{z}$
1, 11	12	$\frac{bee}{z} = z$
12 × z	13	$bee = zz = 28224$
13 ÷ b	14	$ee = \frac{zz}{b} = 1024$
1 ω 2	15	$e = \sqrt{1024} = 32$
1 — 15	16	$a + y = 136$
16 ◎ 2	17	$aa + 2ay + yy = 18496$
$4 \times \overline{4}$	18	$+ 4ay = 4ee = 4096$
17 — 18	19	$aa - 2ay + yy = 14400$
19 ω 2	20	$a - y = \sqrt{14400} = 120$
$16 + \underline{20}$	21	$2a = 136 + 120 = 256$
21 ÷ 2	22	$a = 128$
16 — 22	23	$y = 136 - 128 = 8$

Hence it appears, that 8. 32. 128. are the three Numbers required.

Proof, $8 : 32 :: 32 : 128$
And $8 + 32 + 128 = 168$ &c.

Queſt. 12. A Gentleman diſtributes a Number of Pence in ÷ amongſt four poor People; ſo that if the Numbers of each Perſon's Pence were multiplied continually together, the Total (or *Laſt*) Product of them all would be $109395\frac{9}{16}$; or in Decimals, 109395,5625. And the two leaſt Numbers being multiplied together, their Product would be 36,75. 'Tis required to tell the whole Sum, and each Perſon's part of it.

Let *a*, *e*, *y*, *u*, repreſent the four Numbers.

Then	1	$a \times e \times y \times u = aeyu = 109395{,}5625 = z$
	2	$ae = 36{,}75 = p$ by the Queſt.
	3	$a : e :: e : y :: y : u$ by the Queſt.
3 ∵	4	$ue = yy$
	5	$ay = ee$
5 ÷ *a*	6	$y = \frac{ee}{a}$
6 ⊖ 2	7	$yy = \frac{eeee}{aa}$
4, 7	8	$ue = \frac{e^4}{aa}$
8 ÷ *e*	9	$u = \frac{e^3}{aa}$, for $\frac{e^4}{aa} \div e = \frac{e^4}{aae} = \frac{e^3}{aa}$
1, 6, 9	10	$a \times e \times \frac{ee}{a} \times \frac{e^3}{aa} = z$
10,	11	$\frac{e^6}{aa} = z$
viz.	12	$\frac{e^6}{z} = aa$. for $e^6 = zaa$. Ergo, &c.
2 ⊖ 2	13	$aaee = pp$
13 ÷ *ee*	14	$\frac{pp}{ee} = aa$
13, 14	15	$\frac{pp}{ee} = \frac{e^6}{z}$
15 ×	16	$e^8 = zpp = 147745544{,}37890625$
16 ɯ 2	17	$e^4 = \sqrt{zpp} = 12155{,}0625$
17 ɯ 2	18	$ee = \sqrt{:\sqrt{zpp}} = 110{,}25$
18 ɯ 2	19	$e = \sqrt{:\sqrt{:\sqrt{zpp}}} = 10{,}5$
2 ÷ 19	20	$a = 3.5$
18 ÷ 20	21	$y = 31.5$
9,	22	$u = \frac{e^3}{aa} = \frac{1157{,}625}{12{,}25} = 94{,}5$

From the four laſt Steps it appears, that the Sum he diſtributed was 140 Pence = 11 *s.* 8 *d.* *viz.* $3,5 + 10,5 + 31,5 + 94,5 = 140$.

I have made choice of theſe few plain and familiar Queſtions, purely for the young Learner's Exerciſe, and have placed them in ſuch Order, as may lead him gradually into this moſt excellent Art of Equations. I have likewiſe traced them from Step to Step, thereby to explain and render each Proceſs eaſy (even to the meaneſt Capacity) which I doubt not but with a little Conſideration will eaſily be underſtood; and then he may proceed to the Reſolving others, of a different and more difficult nature, of which there are great varieties in ſeveral Authors, to whom I refer thoſe that are deſirous of ſuch Speculations; and proceed (according to my Deſign) to Extracting the Roots (in Numbers) of all pure Equations; then to the Reſolving of ſuch Problems only, as will raiſe different Degrees of Mixt or Affected Equations; and there ſhew how the ſame may be reſolved into Numbers.

Which laſt Work hath heretofore been a Buſineſs of great Difficulty and Labour; as the Studious in this Art no doubt have experienced. But the ſame is now rendered much eaſier, and made practicable, by a new Method of Converging Series, invented by the Learned and Ingenious Mr. *Joſeph Raphſon*, Fellow of the *Royal Society*; and publiſhed in a *Latin* Treatiſe, by the Title of *Analyſis Æquationum Univerſalis*, printed 1690: wherein he hath given (and demonſtrated) general Theorems for the Solving (or Extracting the Roots) of all manner of Equations, howſoever Mixt or Affected.

I ſhall here give a ſmall Abſtract thereof, and refer the more inquiſitive Reader to the Book itſelf; where (I doubt not, but) he will meet with ample Satisfaction.

⁂

CHAP.

CHAP. IX.

An Abſtract of Mr. Raphſon's *Method of Converging Series.*

IN this Abſtract, I ſhall paſs over the Author's Demonſtration, giving only two Examples of the Method, explaining the ſame; then inſert a few of his Theorems, with ſome Remarks upon the whole, and ſo conclude it.

Example in the Square Root.

Viz. $aa = c \left\{ \begin{array}{l} \text{Any given Number whoſe Square} \\ \text{Root is required.} \end{array} \right.$

What is the Value of the Root a?

Let the Root a be ſuppoſed to be divided into two Parts; to expreſs each of which, put or ſubſtitute two Letters, *viz.* $g + x = a$;

Then is, $gg + 2gx + xx = aa = c$.

He rejecteth the Power of x, (viz. xx) and then it becomes $gg + 2gx = c$; which is $2gx = c - gg$, and from hence ariſeth this

$$\textit{Theorem}, x = \frac{c - gg}{2g}$$

Note, The Number ſignified by g, may be taken at pleaſure; but if g be made the firſt Root, or ſingle ſide of the given Square, the ſooner will the other Member, (or Converging part x) converge to the true Root.

To explain what is here ſaid, take this Example:

$aa = c = 2$. *As in* Analyſis Æquationum, *pag.* 5.

take the firſt $g = 1$.

$$\text{Then} \left\{ \begin{array}{r} c = 2 \\ -gg = 1 \end{array} \right\} = 1$$

$$\text{And } 2g = 2) \quad \underline{} \quad 1,0 \; (,5 = x$$

$$\begin{array}{rl} \text{To the Firſt } g = 1, \text{ add} & ,5 = x \\ + & \underline{,5} \\ \text{the new } g = & 1,5 \text{ for a Second Operation.} \end{array}$$

$$\text{Then} \left\{ \begin{array}{r} c = 2 \\ -gg = \underline{2,25} \end{array} \right\} = -0,25$$

$$\text{And } 2g = 3) \quad \underline{-0,25} \quad (-,083 = x.$$

Let $g = 1,5$ from which muſt now be taken the Converging Number x, becauſe it hath the Sign $-$ prefix'd to it.

$$\begin{array}{rl} & 1,5 \\ & \underline{-\ ,083} \\ \text{the new } g = & 1,417 \text{ for a Third Operation.} \end{array}$$

$$\begin{array}{l} \text{Then } c = 2 \\ \quad -\underline{gg} = \underline{2,007889} \\ \text{and } 2g = 2,834) \ \underline{-\ ,007889} \ (-,002783 = x \end{array}$$

$$\begin{array}{rl} & 1,417 \\ & \underline{-\ ,002783} \\ a = & 1,414217 \end{array}$$

Or if more Accuracy be required, it may be call'd a new g, for a Fourth Operation; and by repeating the Operations, you may have as many Places in the Root as you pleaſe.

Next, for the Extraction of the Cube Root, *viz.*

$$aaa = d,$$

he puts $g + x = a$, as before in the Square.

Then is, $ggg + 3ggx + 3gxx + xxx = aaa$

And if the Powers of x, viz. $3gxx + xxx$, be rejected, or caſt off, as before in the Square, (the which he doth in all Equations) then it will become

$$ggg + 3ggx = aaa = d, \text{ and } 3ggx = d - ggg$$

from hence will ariſe this *Theorem* $x = \dfrac{d - ggg}{3gg}$.

Let $d = 37945$, as in *Analyſis Æquationum*, pag. 6,

$$3\dot{7}94\dot{5} = d$$

take the firſt $g = 3$, and $d = 37$, that is to the firſt Point.

Then $\left\{\begin{array}{r} d = 37 \\ -ggg = 27 \end{array}\right\} = 10$, &c.

and $3gg = 27)\quad 100\quad (03 = x$

Firſt, $g = 3$, to which add the Converging x,

$+03$

the new $g = 33$ for a ſecond Operation.

Then $d = 37945$

$-ggg = 35937$

and $3gg = 3267)\quad 2008;0\ (;61 = x$

$33{,}$

$+\ {,}61$

the new $g = 33{,}61$ for a Third Operation.

Then $\left\{\begin{array}{r} d = 37945 \\ -ggg = 37966;934881 \end{array}\right.$

and $3gg = 3388;8963)\ -21{,}934881\ (-{,}006472 = x$

Laſtly $g = 33{,}61$ from which muſt now be taken the $-\ {,}006472 = x$

hence, $a = 33{,}603528$

Or if more Exactneſs be required, then it may be called a new g, as before, &c.

I ſhall deſiſt giving more Examples in Numbers, this being ſufficient to ſhew the Method of proceeding in each Equation; and only inſert a Table or two of the ſeveral Theorems, as they are recorded in Mr. *Raphſon*'s Book before mention'd.

The given Powers, whoſe Root is required.	Firſt Theorems.	Second Theorems.
For the Square Root $aa = c$	$x = \frac{c - gg}{2g}$	$a = \frac{c + gg}{2g}$
For the Cube $aaa = d$	$x = \frac{d - ggg}{3gg}$	$a = \frac{d + 2ggg}{3gg}$
For the Biquadr. $aaaa = f$	$x = \frac{f - gggg}{4ggg}$	$a = \frac{f + 3gggg}{4ggg}$
For the Surſolid $a^5 = f$	$x = \frac{f - ggggg}{5gggg}$	$a = \frac{f + 4g^5}{5gggg}$

By what is here ſet down, it is obvious how to proceed for Extracting the Root (by this Method) of any ſimple Power, not only of theſe here inſerted, but of any how high ſoever it be.

A Table of Theorems, for the Reſolving of Adfected Equations, *wherein I retain only thoſe with the Converging* (x) *and have omitted the ſecond ſort, which Converge by the* (a) *it ſelf; ſuch as in the ſecond ſort of the foregoing Table.*

Equations propoſed.	Theorems.
$aa + ba = c$	$x = \frac{c - gg - bg}{2g + b}$
$aa - ba = c$	$x = \frac{c + bg - gg}{2g - b}$
$ba - aa = c$	$x = \frac{c + gg - bg}{b - 2g}$
$aaa + ca = d$	$x = \frac{d - ggg - cg}{3gg + c}$
$aaa - ca = d$	$x = \frac{d + cg - ggg}{3gg - c}$
$ca - aaa = d$	$x = \frac{d + ggg - cg}{c - 3gg}$
$aaa + baa + ca = d$	$x = \frac{d - ggg - bgg - cg}{3gg + 2bg + c}$
$aaa + baa - ca = d$	$x = \frac{d - ggg - bgg + cg}{3gg + 2bg - c}$
$aaa - baa - ca = d$	$x = \frac{d + bgg + cg - ggg}{3gg - 2bg - c}$
$baa + ca - aaa = d$	$x = \frac{d + ggg - bgg - cg}{2bg + c - 3gg}$

This Table is continued in Mr. *Raphſon*'s Book to four or five Pages more, but for brevity ſake I am forced to omit inſerting them.

There be ſeveral things obſervable in this Method; amongſt the reſt, theſe are not the leaſt, *viz.* That at each Operation, the Converging Number x will double the laſt preceeding g, (or Number of Figures in the laſt Root, *eſpecially after the ſecond Operation*) the Imperfection being only in the laſt Figure of the Root, ſo increaſed, which often proves too large; and therefore conſequently the next Converging Number x, will have the Negative Sign —.

Alſo if there happen to be a Miſtake committed in any Operation, ſuch Miſtake doth not deſtroy the preceeding Work, for the ſame will be rectified (though it be not diſcovered) in the next ſucceeding Operation, unleſs it be very groſs.

Again, it produceth the Roots of all Powers, be they never ſo high, in the ſame manner, and with the ſame exactneſs, as it doth thoſe of a lower Rank, the reſpective Involutions being conſidered, which require to be always of the ſame height with the given Powers, and the Diviſor of the next inferior or lower Powers.

Theſe high Involutions and large Diviſors, have often cauſed me to wiſh, that ſome means might be uſed of improving this Method (or ſome lucky Diſcovery made of another) ſuch as would produce the ſeveral Reſolvends and Diviſors in lower Terms: And being (upon an extraordinary occaſion) imployed in the Calculation of ſeveral Problems, conſiſting of very high Adfected Equations, made me the more deſirous thereof, and put me upon thoughts, how (if poſſible) to accompliſh the ſame; which if effected, would undoubtedly render the ſeveral Operations more eaſy.

Whilſt I was ruminating upon this, there came into my mind the following Method, the which hath not only anſwer'd my Deſire in that particular, of producing low Reſolvends and Diviſors, but even out-done my Expectation, in cauſing the Root to converge quicker: for at each Operation, it will in moſt caſes triple the Places of Figures in the laſt preceeding Root; in ſeveral it will do more, and in ſome caſes, the Root convergeth by a continued Series, without repeating the Operations at all, as will plainly appear in the Practice thereof.

CHAP. X.

Containing a new and general Method of Extracting the Roots (in Numbers) of all pure Equations.

I SHALL communicate this Method with all the Plainneſs and Brevity I can, and therein ſhew the Proceſs uſed in raiſing and forming the Theorems for the Solution of each Equation, as they come in their Order.

Now becauſe there is required an Involution of the increaſed Root, at the repeating of ſeveral Operations, (tho not ſo high as the given Power) I will here inſert an eaſy Method of compoſing the Square of an increaſed Root, by having the Square and its Root, of one part thereof, with the increaſing Numbers of the other part of the Root, perform'd without involving the whole Root ſo increaſed.

```
                    Root = .7  4  9  3  8  0  5  2
         Square of 7 = 49|1 6
                          2 8
                          2 8
         Square of 74 = 5476|8109
                              2247
                              2247
                             666
                             666
       Square of 7493 = 56145049|64002504
                                 149876lo
                                 14987610
                                 3746900
                                 3746900
                               59944
                               59944
  Square of 74938052 = 5615711637554704
```

This is ſo obvious, I ſuppoſe there needs no Explanation thereof: *The Difference betwixt it and the common Method, I leave to the Conſideration of the Ingenious.*

From the foregoing Geneſis, or Compoſition of the Square, it is evident, that for each ſingle Figure (or Cypher) in the Root, there will ariſe two places of Figures in the Square; and from the like Compoſition in the Cube, it would be as evident there will ariſe Three, and in the Biquadrate Four, &c.

That is, there will ariſe ſo many Places of Figures in the Involved Number, for one in the Root, as the Index of ſuch Power denotes.

Therefore when any Number is propoſed to have its Root extracted, the preparative Work is to conſider the Index of the required Root, and to allow ſo many Places of Figures in the given Number, for each ſingle Figure in the Root, as ſuch Index ſhall denote; which Account muſt always take its beginning from the place of Unity, and aſcend towards the left hand, if the given Number be Integers, or deſcend towards the Right in Decimal Parts.

Of the Index of any Root, ſee *Pag.* 13. and ſuch Index is the ſame with that of the Power, *viz.*

$$\left.\begin{aligned} aa &= b \\ aaa &= c \\ aaaa &= d \\ aaaaa &= f \end{aligned}\right\} \text{is} \left\{\begin{aligned} a^2 &= b \\ a^3 &= c \\ a^4 &= d \\ a^5 &= f \end{aligned}\right\} \text{then} \left\{\begin{aligned} a &= \sqrt[2]{b} \\ a &= \sqrt[3]{b} \\ a &= \sqrt[4]{d} \\ a &= \sqrt[5]{f} \end{aligned}\right.$$

And from hence is deduced the Method of diſtinguiſhing (or rather parting) the given Numbers with Points, ſet over, or under their proper Figures.

For Example, Suppoſe any Number given,

As, 56430145789,723

out of which, if it be required to extract any of theſe Roots, it muſt be pointed (according to the foregoing Conſiderations) in this manner; that is, for the

Square

Square Root	$\dot{5}6\dot{4}3\dot{0}1\dot{4}5\dot{7}8\dot{9},7\dot{2}3\dot{0}$
Cube Root	$5\dot{6}43\dot{0}14\dot{5}78\dot{9},72\dot{3}$
Biquadrate Root,	$56\dot{4}301\dot{4}578\dot{9},723\dot{0}$
Surſolid Root,	$\dot{5}6430\dot{1}4578\dot{9},7230\dot{0}$

And by theſe Points is known how many Places of Figures there will be in the Root ſought; for ſo many as are the Points, ſo many muſt be the Places of Figures in the Root. Now whether they be Integers or Decimals, is eaſily determined, by the Places of the Points over each. Theſe premiſed, we may proceed to Examples in each.

Note, That in all the Theorems for reſolving of Equations (or extracting of Roots) by the following Method, I make uſe of theſe Letters (G) (r) (e) (D) to repreſent each diſtinct part thereof, *viz.*

G, Always denotes the given Reſolvend, or known Side of the Equation.

r, Repreſents the Root firſt taken, and when increaſed, *&c.*

e, Repreſents the Converging Part of the Root; by which, r is either increaſed or diminiſh'd, according as its Sign denotes, to wit, $+e$, or $-e$.

D, Is put for the Dividend, which is produced from G, being divided; and leſſen'd by r, (into the Coefficients of Adfected Equations) according as the Root requires: all which will plainly appear, and be eaſily underſtood, as we proceed on in the Examples.

I. *For the Extracting of the Square Root,* viz. $aa = G$. What is the Value of a?

In this Method (as in all others whatſoever) we ſuppoſe the Root a to be divided in two parts; to expreſs which, put the two Letters before mention'd.

That is,	1	$r + e = a$
$1 \circledcirc 2$	2	$rr + 2re + ee = aa = G$
$2 \div 2$	3	$\frac{1}{2}rr + re + \frac{1}{2}ee = \frac{1}{2}G$
$3 - \frac{1}{2}rr$	4	$re + \frac{1}{2}ee = \frac{1}{2}G - \frac{1}{2}rr = D$

And from hence will ariſe this

Theorem $\left\{ \frac{\mathcal{D}}{r+\frac{1}{2}e} = e. \right.$

First take an Example of a true Square Number.

$$\text{Let } aa = \dot{4}1\dot{9}6\dot{8}3\dot{7}0\dot{8}9 = G$$

$r = 6$, That is, the First Single Root (or Side) to 41

$$2\dot{0}98\dot{4}18\dot{5}44\dot{,}5 = \tfrac{1}{2}G$$
$$-\tfrac{1}{2}rr = -18$$
$$)\ 298418544,5 = \mathcal{D}\ (4 = e$$
$$248 = re + \tfrac{1}{2}ee$$
$$50418\quad (7 = e,$$
$$45045 = re + \tfrac{1}{2}ee$$
$$53735\quad (8 = e$$
$$51792 = re + \tfrac{1}{2}ee$$
$$194344,5\quad (3 = e$$
$$194344,5 = re + \tfrac{1}{2}ee$$
$$(0,0)$$

$r = 6$
$+e = 4$
$r + e = 64783$

but $r + e = a$
therefore
$a = 64783$

I preſume the whole Proceſs of this is plain; however, for the benefit of ſuch as do not underſtand an Algebraick Theorem, let them take it in words thus.

Firſt, Point the Reſolvend (or given Number) as before taught; then take the greateſt Square to the firſt Point thereof, out of the Reſolvend to that Point: after which, take half the remainder of the Reſolvend, and point it a-new; then make the Root of that Square ſo taken, a Diviſor; inquiring how oft it may be found in the new Dividend, to the next Figure forward, reſerving that Figure under the next Point, for the half Square of the Quotient Figure: which being found, and ſo placed, multiply the Diviſor by that Quotient, adding in the Tens (if any aroſe from the half Square of the Quotient), as in plain Diviſion, &c. Then annex the Quotient to the laſt Diviſor, thereby making a new Diviſor, with which proceed in all reſpects as with the laſt; and ſo on, until all be finiſhed.

If any ſeeming Difficulty appear, 'tis only in the true placing of the half Square of the Quotient Figure when it proves an odd Number: in that caſe, bring down one Figure more of the next Period; under which, place the odd 5, which will always ariſe from the halving of the Square of an odd Number.

As for inſtance, Suppoſe 7, the Square thereof is 49, the half of which is 24,5 to be placed as in the Second Operation of the laſt Example.

Note, That if the given Number be a *Surd*, the Root *r* will become a continued Series *ad infinitum*; if it be ſtill increaſed with the Converging Number *e*.

But if the Number of Places (in the Root of a *Surd* Number) be limited, the Root *r* needs not be increaſed with *e*, to the laſt Operation of the whole Proceſs; for the Work may be abbreviated, as in this Example. Suppoſe the Square Root of 3, (a *Surd* Number) were required to ſixteen places of Figures therein.

Equation $aa = G$, and $G = 3$.

$$r = 1, \qquad 1,5 = \tfrac{1}{2}G$$

$$+e = ,7 \qquad -0,5 = \tfrac{1}{2}rr$$

$$r + e = 1,7320508)\ 1,0000000000000000 = \mathfrak{D}$$

$$\underline{945} = re + \tfrac{1}{2}ee \qquad (,7 = e$$

$$550 \qquad (3 = e$$

$$\underline{5145} = re + \tfrac{1}{2}ee$$

$$3550 \qquad (2 = e$$

$$\underline{3462} = re + \tfrac{1}{2}ee$$

$$880000 \qquad (05 = e$$

$$\underline{8660125} = re + \tfrac{1}{2}ee$$

$$139875000 \qquad (08 = e$$

$$\underline{138564032} = re + \tfrac{1}{2}ee$$

$$1310968$$

Having thus increaſed the Root to 8 or 9 Places by the Converging *e*, the reſt may be obtained by plain *Diviſion*, making the Root ſo found, a Diviſor; but I alſo contract plain Diviſion, by omitting (or pricking off) one Figure of the Diviſor at each Operation; and then the Work will ſtand thus:

$$\begin{array}{r|l}
r = 1{,}7320508)\ 131096800 & (07568877 = e \\
\cdots\cdots\quad \underline{121243556} & \\
9853244 & \\
\underline{8660250} & \\
1192994 & \\
\underline{1039230} & \\
153764 & \\
\underline{138560} & \\
15204 & \\
\underline{13856} & \\
1348 & \\
\underline{1211} & \\
137 & \\
\underline{119} & \\
\&c. &
\end{array}$$

This Method of Contracting the Diviſor, is of ſingular uſe upon ſeveral Occaſions, where it may be admitted.

And what is here done as to this Abridgement, may in like manner be performed in the Cube, Biquadrate, &c.

$$\begin{array}{rl}
\text{Laſt } r = & 1{,}7320508 \\
+\,e & \underline{\qquad\qquad 07568877} \\
a = & 1{,}732050807568877
\end{array}$$
as was required.

II. *Of Extracting the Cube Root.*

Equation $aaa = G$. To find the Value of a?

$$\begin{array}{c|c|l}
\text{Put} & 1 & r + e = a \\
1 \circledcirc \underline{3} & 2 & rrr + 3rre + 3ree + eee = aaa = G \\
2 \div 3 & 3 & \frac{1}{3}rrr + rre + ree + \frac{1}{3}eee = G \\
3 \div r & 4 & \frac{1}{3}rr + re + ee\,\frac{1}{3}\,\frac{eee}{r} = \frac{1}{3}G \div r.
\end{array}$$

Let $\frac{1}{3}eee \div r$ be rejected, or caſt off, as being of ſmall Value, then it will become

$$\frac{1}{3}rr + re + ee = \frac{1}{3}G \div r.$$

and thence will $re + ee = \dfrac{\frac{1}{3}G}{r} - \frac{1}{3}rr = \mathfrak{D}$.

Hence ariſeth this

$$\text{Theorem}\left\{\frac{\mathfrak{D}}{r+e} = e.\right.$$

Let $37945 = G$, (as in *Pag.* 56. of this Tract.)

then $r = 30$, and $12648,335 = \frac{1}{3}G$ . . . . . .

$421,6111 = \frac{1}{3}G \div r$

$r = 30$ $\qquad - 300, = \frac{1}{3}rr$

$+e = 3$

$r + e \quad 33,6$ $\qquad 121,6111 = \mathcal{D}$ $\qquad (3 = e$

$99 = re + ee$

2261 $\qquad (,6 = e$

$2016 = re + ee$

New $r = 33,6$ $\qquad 376,43849206 = \frac{1}{3}G \div r$

$- 376,32 = \frac{1}{3}rr$

$r + e = 33,6035262)$ $\quad ,11849206 = \mathcal{D}$ $\qquad (,003 = e$

$100809 = re + ee$

1768306 $\qquad (5 = e$

$1680175 = re + ee$

$a = 33,6035262$, &c. $\qquad 8813100$ $\qquad (2 = e$

$6720704 = re + ee$

Here you have the Cube Root extracted to Nine Places, at two eaſy Operations, the Defect being but Unity, in the laſt place thereof.

209239600 $\qquad (6 = e$

201621156

761844400 $\qquad (2 = e$

672070524

This Method doth alſo converge with the Negative Sign —, if the Value of *e* happen to be taken too large; but to prevent that, take but one Value of *e*, at the firſt Operation, and but four at the ſecond; by ſo doing, the Root will have but ſix places at two Operations: But then a third will produce ſeventeen Places true; each Operation converging gradually, with the Affirmative Sign, (which I take to be beſt.)

Take another Example of a true Cube Number.

$aaa = 48627125 = G$

$r = 300$ $\qquad 16209041,66$ &c. $= \frac{1}{3}G$

$54030,1588$ &c. $= \frac{1}{3}G \div r$

$- 30000 = \frac{1}{3}rr.$

$r + e = 366,$ $\qquad 24030,1388 = \mathcal{D}$ $\qquad (66 = e$

$216 = re + ee$

2430

New $r = 366$ $\qquad 2196 = re + ee.$

But, as was ſaid above, I take but one Place, then

$$\begin{array}{ll}
r = 360 \quad \text{and} & 45025,1157 = \tfrac{1}{3}G \div r \\
 & \underline{-\,43200, = \tfrac{1}{3}\,rr} \\
r + e = 365,0003 & 1825,1157 = \mathcal{D} \\
 & \underline{1825, = re + ee} \;(5,0003 = e \\
 & 0,11570000 \\
\text{hence } a = 365,0003 & 10150009 = re + ee
\end{array}$$

The Root is really but 365; this Overplus of 0003, would at the next Operation vaniſh. And from this Example, 'tis apparent, that four Places will truly ariſe in the Root, at the ſecond Operation.

The Work of theſe two laſt Examples, (I preſume) is ſo plain and clear, that it's needleſs to expreſs the Proceſs thereof in Words: Alſo from hence it's as plain, that the Extraction of the Cube Root (heretofore ſo very difficult) is by this Method render'd eaſier than that of the Square, as it's uſually perform'd.

III. *Of Extracting the Biquadrate Root*: viz.

$aaaa = G$. What is the Value of a?

Let $aaaa = G = 2741583974$, and $r + e = a$

$$\begin{array}{l|l|l}
\text{Then} & 1 & rrrr + 4rrre + 6rree + 4reee + eeee = G \\
1 \div 4 & 2 & \tfrac{1}{4}rrrr + rrre + 1\tfrac{1}{2}rree + reee + \tfrac{1}{4}eeee = \tfrac{1}{4}G \\
2 \div rr & 3 & \tfrac{1}{4}rr + re + 1\tfrac{1}{2}ee + \dfrac{eee}{r} + \dfrac{4eee}{rr} = \tfrac{1}{4}G \div rr
\end{array}$$

Let $eee \div r$, and $\tfrac{1}{4}eeee \div rr$ be rejected, or caſt off, (as before) then it will become

$$\tfrac{1}{4}rr + re + 1\tfrac{1}{2}ee = \tfrac{1}{4}G \div rr$$

and conſequently $re + 1\tfrac{1}{2}ee = \dfrac{\tfrac{1}{4}G}{rr} - \tfrac{1}{4}rr = \mathcal{D}$.

From hence ariſes this

$$\textit{Theorem}\left\{\frac{\mathcal{D}}{r + 1\tfrac{1}{2}e} = e.\right.$$

First $r = 200$

$r + e = 229$
$+ \frac{1}{2}e = 14{,}5$
Diviſor 23
Diviſor $233{,}5$

New $r = 229,$
$- 1\frac{1}{2}e = {,}15$
Diviſor $228{,}85$
New $r = 228{,}9$
$- 1\frac{1}{2}e = {,}105,$
Diviſor $228{,}795,$
New $r = 228{,}83$
$- 1\frac{1}{2}e = {,}009$
Diviſor $228{,}821$

$$\begin{array}{r} r = 229, \\ - \quad {,}1765 \\ \hline a = 228{,}8235 \end{array}$$

$2741583974 = G$
$685395993{,}5 = \frac{1}{4}G$
$17134{,}899$ &c. $= \frac{1}{4}G \div rr$
$- 10000 = \frac{1}{4}rr$
$7134{,}89$ &c. $= \mathcal{D}$ $(29 = e$
$46 = re + 1\frac{1}{2}ee$
$2534{,}8$
$2101{,}5 = re + 1\frac{1}{2}ee.$
$13069{,}8498 = {}^{1}G \div rr$
$- 13110{,}25 = \frac{1}{4}rr$
$- 40{,}4002 = \mathcal{D}$ $(-{,}1 = e$
$22{,}885 = re - 1\frac{1}{2}ee$
$17{,}51520$ $(-7 = e$
$16{,}01565 = re - 1\frac{1}{2}e$
$1{,}499550$ $(6 = e$
$1{,}372926 = re - 1\frac{1}{2}e$
1266240 $(5 = e$
1144105

In this Example, it appears I took the ſecond $r = 229$ too big, and therefore in the ſecond Proceſs, the Member e convergeth with the Sign — and conſequently (in this Caſe) the Diviſor muſt be $re - 1\frac{1}{2}e$; alſo at each Diviſion, e is to be taken from the laſt r, and the remainder will be the New r.

IV. *Of Extracting the Root of a Surſolid (or Fifth Power)* viz. $aaaaa = G$. What's the Value of a?

In Numbers $aaaaa = 43467897 = G$.

Put	1	$r + e = a$. Reject all the Powers of e above ee
then	2	$rrrrr + 5rrrre + 10rrree = G$
$2 \div 5$	3	$\frac{1}{5}rrrrr + rrrre + 2rrree = \frac{1}{5}G$
$3 \div r^3$	4	$\frac{1}{5}rr + re + 2ee = \frac{1}{5}G \div rrr$
$4 - \frac{1}{5}rr$	5	$re + 2ee = \dfrac{\frac{1}{5}G}{rrr} - \frac{1}{5}rr = \mathcal{D}$

From hence ariſeth this

Theorem $\left\{ \frac{\mathcal{D}}{r+2e} = e. \right.$

Firſt $r = 30$	$43467897 = G$	
	$8693579,4 = \frac{1}{5}G$	
	$321,96$ &c. $= \frac{1}{5}G \div rrr$	
	$-180, = \frac{1}{5}rr$	
$r + e = 33$	$141,96 = \mathcal{D}$	$(3 = e$
$+ e = 3$	$108 = re + 2ee$	
Diviſor 36		
New $r = 33,$	$241,91160019 = \frac{1}{5}G \div r^3$	
$+ 2e = 1,4$	$-217,8 = \frac{1}{5}rr$	
Diviſor $34,4$	$24,11160019 = \mathcal{D}$	$(,7 = e$
New $r + e = 33,7009$	$24,08 = re + 2ee$	
$+ e = ,0009$	$,03160019$	$(,0009 = e$
Diviſor $33,7018$	$3033162 = re + 2ee$	

$r + e = 33,7009 = a.$

But if more Exactneſs be required, the next Operation will produce 17 Places of Figures true.

I doubt not but by theſe few Examples, the nature of, and manner how to proceed in this Method is ſufficiently cleared; as to the Extraction of Roots out of ſimple or pure Equations, how highly ſoever they be.

And becauſe there is great Care and Trouble attends the continued Involutions of $r+e$; or $r-e$; eſpecially to any conſiderable height, by reaſon of the *Unciæ*, (or Numeral Figures that ariſe by involving the Quantities) I will here ſhew how any Power, of what Degree ſoever, may be ſpeedily and eaſily raiſed, without involving the Quantities at all: Thus,

Firſt make the Index of the higheſt Power of r, the ſame to that of the given Power of a; then the next Power of r, (into e) muſt have its Index leſs by Unity, and the third Power of r (into ee) muſt have its Index leſs than the firſt by 2, that is, their Indices decreaſe in Arithmetical Progreſſion.

Next for obtaining the *Unciæ*, or reſpective Numeral Figures that would ariſe by a continued Involution of

$r+e$, prefix to the ſecond Power of r (into e) the Index of the firſt (or ſimple) Power of r; and to the third Power of r (into ee) prefix half the Product of the firſt Index into the Index of the ſecond, and theſe will be the true Numbers required.

For the clearing of this, take an Example or two: Suppoſe a^9 were given, and you are to raiſe $r+e$ to the ſame height (that is, ſo much thereof as is required in this Method) firſt ſet down r^9, to this add r^8e, with the Index of the firſt prefixed to it; then it will be r^9+9r^8e: To theſe add r^7ee, with half the Product of 9×8, to wit, $9\times4=36$, and then it will be $r^9+9r^8e+36r^7ee=a^9$, as was required.

Again, ſuppoſe a^{18}, were given; to raiſe the reſt as before, $r^{18}+18re^{17}$, are the two firſt Members, and the half of 18×17, that is, $9\times17=153$, and then they will be $r^{18}+18r^{17}ee+153r^{16}ee=a^{18}$.

Thus you may proceed on to what Height, and to as many Terms as you pleaſe, by taking $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, &c. of the *Unciæ* into the Indices they belong to.

And this I take to be a new Diſcovery, having never ſeen any thing performed in this kind, with the like Eaſe and Expedition.

CHAP. XI.

The Reſolving of ſeveral Geometrical Problems, together with the Solution of their Equations in Numbers.

FOR the reſolving of *Geometrical Problems*, it will be requiſite to have a clear Underſtanding of theſe two Propoſitions of *Euclid*, *viz.* the 47th of the 1ſt, and 4th of the 6th Book: of their general (and excellent) Uſe in *Geometry*, (and conſequently in all Parts of the *Mathematicks*) none that hath been converſant therein, but will readily grant; of which *Des Cartes* takes particular notice, in a Letter of his, cited by Dr. *Pell*, in the before-mentioned Book of *Algebra*, *pag*. 65.

And here it may not be amiſs (for the Benefit of the young Learner) to give a particular Account of the aforeſaid Propoſitions.

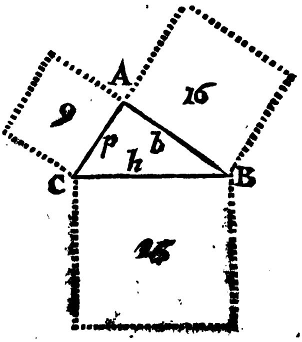
Prop. I. In all Right-angled Triangles, the Square of the Hypotheneuſe (or greateſt Side) is equal to the Sum of the Squares of the Baſe and Perpendicular (or of the other two ſides.) *Eucl*. 1. 47.

Suppoſe the Triangle *CAB*, to be right-angled at *A*: And let

$h = CB$ the Hypotheneuſe.
$b = BA$ the Baſe.
$p = AC$ the Perpendicular.

Then, according to the Propoſition, it will be

$$\left.\begin{aligned} hh &= bb + pp \\ bb &= hh - pp \\ pp &= hh - bb \end{aligned}\right\}$$



To explain, or illuſtrate this noble Propoſition in Numbers,

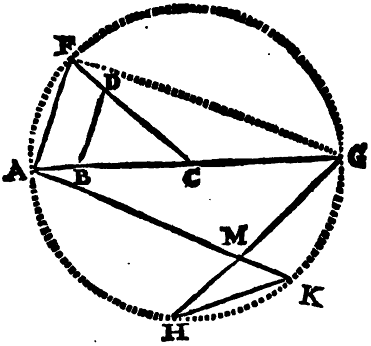
Let $h = 5$. $b = 4$: and $p = 3$. Then will $hh = 25$. $bb = 16$: and $pp = 9$.
But $16 + 9 = 25$.

Prop. II. The Sides of like Triangles are reſpectively proportional to each other. *Eucl*. 6. 4.

That is, if there be two Triangles (either right or oblique) that have each Angle of the one, equal to each Angle of the other, then are thoſe two Triangles alike, and their reſpective Sides proportional.

Angles are equal, that ſtand upon (or have) equal Arches, either both from the Center, or both from the Periphery of a Circle. As for example, Suppoſe the two Triangles to be *ACF*, and *BCD*, having the Angle at *C*, (the Center) common to both, and *BD*, parallel to *AF*:

Then is the Angle at *D*, equal to that at *F*; and conſequently the Angle at *B*, equal to that at *A*: And, according to the Propoſition, *AC* : *AF* :: *BC* : *BD*; and *FC* : *AC* :: *DC* : *BC*, &c.



Again, the Triangles *AMG*, and *HMK*, are alike; for the Angles *A*, and *H*, ſtand both upon the Arch *GK*, and the Angles *G*, and *K*, ſtand both upon *AH*; alſo the Angle at *M*, is alike in both. Therefore, according to the Propoſition,

AG : *HK* :: *AM* : *HM*, and *GM* : *MK* :: *AM* : *MH*, &c.

To theſe two, add, that the Angle at the Center is double to that at the Periphery; that is, the Angle *FCA*, is double to the Angle *FGA*. *Eucl.* 3. 20.

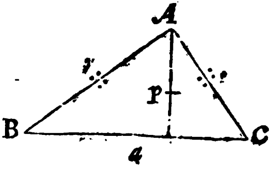
Theſe Propoſitions being premiſed, and pretty well underſtood, the young Algebraiſt may proceed to the Reſolution of Problems, into which I endeavour to introduce him with ſome that be eaſy.

PROB. I.

IN the Right angled Triangle *BAC*, there is given the Sum of the Sides *AB*, and *AC*; alſo the Perpendicular *P*, let fall from the Right Angle *A*, upon the Hypotheneuſe *BC*. To find the Hypotheneuſe, and the Sides.

Let the Hypoth. : $BC = a$. . . $AC = e$. $AB = y$.

Then	1	$e + y = z$ } given { $z = 48$
	2	$p = p$ } given { $p = 17$
	3	$BC = a$, ſought
Prop. 1.	4	$ee + yy = aa$
Prop. 2.	5	$y : a :: p : e$
5.	6	$ye = pa$
1 ⊙ 2	7	$ee + 2ye + yy = zz$,
6 × 2	8	$2ye = 2pa$
4 + 8	9	$ee + 2ye + yy = aa + 2pa$
7, 9,	10	$aa + 2pa = zz$.



This is called the firſt Form of Square adfected Equations, and may be ſolved by compleating the Square; (as in *pag.* 55.) Then it will become

$$aa + 2pa + pp = zz + yp.$$

and conſequently $a + p = \sqrt{zz + pp}$.

Then will $a = \sqrt{zz + pp} - p$

And this Method is eaſy enough, eſpecially by the Method of Extracting the Square Root, (*pag.* 64.) but the ſame may be readier done by the following Method.

Note, *When* Prop. 1. *or* Prop. 2. *is placed in the Margin, they refer to the two Propoſitions before mention'd,* Pag. 71.

The new-found Equation is $aa + 2pa = zz$.

In Numbers $aa + 34a = 2304 = G$.

Put	1	$r + e = a$
1 ⊙ 2	2	$rr + 2re + ee = aa$
1 × 2*p*	3	$2pr + 2pe = 2pa$
3 + 2	4	$rr + 2pr + 2re + 2pe + ee = G$
4 ÷ 2	5	$\frac{1}{2}rr + pr + re + pe + \frac{1}{2}ee = \frac{1}{2}G$
Then	6	$re + pe + \frac{1}{2}ee = \frac{1}{2}G - \frac{1}{2}rr - pr = D$

From hence ariſeth this

$$\textit{Theorem}\left\{\frac{D}{r + p + \frac{1}{2}e} = e\right.$$

L

$r = 30$

$r=30$	$2304=G$	and $17=p$
$\frac{1}{2}rr=450$ } $=-$	$1152=\frac{1}{2}G$	
$pr=510$ }	$960=pr+\frac{1}{2}rr$	
$r+p=47$	$192,00=\mathcal{D}$	$(3,=e$
$+\frac{1}{2}e=1,5$	$145,5$	
Diviſor $48,5$	$46,5000$	$(,9=e$
New $r+p=50,$	$45,405$	
$+\frac{1}{2}e=,45$	$1,0950$	$(2=e$
2 Diviſor $50,45$	$1,0182$	
New $r+p=50,9$	768000	$(1=e$
$+\frac{1}{2}e=,01$	509205	
Diviſor $50,91$	* 258795	$(50823=e$
New $r+p=50,92$	254605	For finding the firſt r.
$+\frac{1}{2}e=,0005$	4190	
Diviſor $50,9205$	4072	Suppoſe $r=4$
New $r+p=50,921$	118	$rr=16$
	100	$3r=12$
* Here I deſiſt forming	18	$28>23$
a new Diviſor.	15	Ergo, $r<4$
Firſt $r=30$	&c.	

$+e=3,921508\,23$

$a=33,92150823$ This is alſo a continued Series, as the Square Root was.

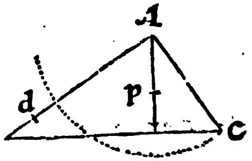
PROB. II.

SUppoſe in the Triangle *BAC*, there were given $d=$ the difference of the Sides *AB* and *AC*. Alſo the Perpendicular *P*, let fall from the Right Angle *A*, upon the Hypothenuſe *BC*. Thence to find the Hypothenuſe, &c.

Let $BC=a$. $AC=e$. And $AB=y$. (as before.)

Then

Then	1	$y - e = d$	given $d = 14$
	2	$p = p$	$p = 20$
	3	$BC = a$ ſought.	
Prop. 1.	4	$yy + ee = aa$	
Prop. 2.	5	$y : a :: p : e$	
5 ∵	6	$ye = pa$	
6 × 2	7	$2ye = 2pa$	
1 ⊙ 2	8	$yy - 2ye + ee = dd$	
8 + 7	9	$yy + ee = dd + 2pa$	
4, 9	10	$aa = dd + 2pa$	
Then	11	$aa - 2pa = dd$	



This is called the Second Form of Square Adfected Equations.

And may be ſolved by Compleating the Square, as in the firſt Form, reſpect had to the Sign —

Then it will become

$$aa - 2pa + pp = dd + pp.$$

And conſequently, $a - p = \sqrt{dd + pp}.$

Then will $a = \sqrt{dd + pp}. + p.$

But this Form may be ſolved (by my Method) as was the laſt, to wit, by a continued Series.

Equation $aa - 2pa = dd.$

In Numbers, $aa - 40a = 196 = G.$

Put	1	$r + e = a$
1 ⊙ 2	2	$rr + 2re + ee = aa$
1 × 2p	3	$2pr + 2pe = 2pa$
2 − 3	4	$rr + 2re - 2pr - 2pe + ee = G.$
4 ÷ 2	5	$\frac{1}{2} rr + re - pr - pe + \frac{1}{2} ee = \frac{1}{2} G$
Then	6	$re - pe + \frac{1}{2} ee = \frac{1}{2} G + pr - \frac{1}{2} rr = D$

From hence ariſeth this

Theorem $\left\{ \frac{D}{r - p + \frac{1}{2}e} = e \right.$

$r > 30$, becauſe of $-2pr$. Suppoſe $r = 30$. Then $rr = 900$, and $2pr = 1200$. But 1200 cannot be taken out of 900, therefore $r > 30$. Suppoſe $r = 50$, $rr = 2500$, and $2pr = 2000$, then $rr - 2pr = 500$, too much; Ergo $r > 30$, but $r > 50$.

Let $r = 40$	$196 = G$	and $20 = p$
$+pr = 800$ }	$98 = \frac{1}{2}G$	
$-\frac{1}{2}rr = 800$ }	$00 = pr - \frac{1}{2}rr$	
$r - p = 20,$	$98, = D$	(4 $= e$
$+\frac{1}{2}e = 2,$	88,	
Diviſor 22,	10,00	(,4 $= e$
$r - p = 24,$	9,68	
$+\frac{1}{2}e = ,2$	32000	(1 $= e$
$r - p = 24,4$	24405	
$+\frac{1}{2}e = ,005$	759500	(3 $= e$
$r - p = 24,41$	732345	
$+\frac{1}{2}e = ,0015$	* 27155	(11124 $= e$
$r - p = 24,413$ *	24413	
* Here I deſiſt forming	2742	
a new Diviſor.	2441	
	301	
Firſt $r = 40$	244	
$+e = 4,41311124$	57	
$a = 44,41311144$	48	
	9	
	8	
	&c.	

PROB. III.

IN the Right-angled Triangle *BCA*, the Baſe (or Side) *BA*, is given; and the Hypothenuſe with the Area, in one Sum: thence to find the Perpendicular *AC*.

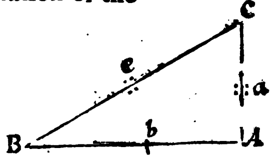
Let $AC = a$, $BC = e$, and $BA = b$, given.

Then	1	$\frac{1}{2}ba =$ the Area.
And	2	$e + \frac{1}{2}ba = z$ the $\left\{\begin{array}{l}\text{Sum of the Hypoth.}\\ \text{and Area.}\end{array}\right.$
Prop. 1.	3	$ee = bb + aa$
$2 - \frac{1}{2}ba$	4	$e = z - \frac{1}{2}ba$
4 Ⓖ 2	5	$ee = zz - zba + \frac{1}{4}bbaa$
3, 5	6	$bb + aa = zz - zba + \frac{1}{4}bbaa$
6 ±	7	$aa - \frac{1}{4}bbaa + zba = zz - bb$

Let $z = 69$, and $b = 7$, then the Equation in Numbers will be $42,93\,a - aa = 418,844.4$

This is a Square Adfected Equation of the Third Form.

Let it be made $ba - aa = G$. This Form uſed to be Reſolved, by compleating the Square, (as before) but firſt the Equation is ſuppoſed to be changed, and made



$$aa - ba = -G.$$

Then $aa - ba + \frac{1}{4}bb = \frac{1}{4}bb - G$.

And $a - \frac{1}{2}b = \sqrt{\frac{1}{4}bb - G}$. Hence, $a = \frac{1}{2}b \pm \sqrt{\frac{1}{4}bb - G}$.

But I ſhall ſolve this by my Method,

Thus,	1	$r + e = a$
1 Ⓖ 2	2	$rr + 2re + ee = aa$
1 × b	3	$br + be = ba$
3 − 2	4	$br - rr + be - 2re - ee = ba - aa = G$
4 ÷ 2	5	$\frac{1}{2}br - \frac{1}{2}rr + \frac{1}{2}be - re - \frac{1}{2}ee = \frac{1}{2}G$

Then will $\frac{1}{2}be - re - \frac{1}{2}ee = \frac{1}{2}G + \frac{1}{2}rr - \frac{1}{2}br = D$.

From hence ariſeth this

$$\textit{Theorem}\left\{\frac{D}{\frac{1}{2}b - r - \frac{1}{2}e} = e.\right.$$

$$\dot{4}2,\dot{9}3 = b \qquad \dot{4}1\dot{8},\dot{4}4\dot{4}4 = G.$$

If $r = 20$, then $rr = 400$, and $br = 858,6$

But $858,6 - 400 = 458,6$ *Ergo*, $r < 20$.

Let

Let $r = 10$		
$+\frac{1}{2}rr = 50$	$418,8444 = G.$	
$-\frac{1}{2}br = 214,65$	$209,4222 = \frac{1}{2}G$	
$-164,65 =$	$164,65 = \frac{1}{2}rr - \frac{1}{2}br$	
$\frac{1}{2}b - r = 11,465)$	$44,7722 = \mathcal{D}$	$(4 = e$
$-\frac{1}{2}e = 2$	$37,860$	
Diviſor $9,465$	$6,9122$	$(,9 = e$
	$6,3135$	
New $r = 14,$	59870	$(9 = e$
$\frac{1}{2}b - r = 7,465$	58680	
$-\frac{1}{2}e = ,45$		
	119000	$(1 = e$
Diviſor $7,015$	64745	
New $r = 14,9$	$* 542550$	$(838 = e$
$\frac{1}{2}b - r = 6,565$	517960	
$-\frac{1}{2}e = ,045$	24590	
Diviſor $6,520$	19422	
New $r = 14,99$	5168	
$\frac{1}{2}b - r = 6,475$	5176	
$-\frac{1}{2}e = ,0005$	&c.	
Diviſor $6,4745$		

* Here I deſiſt forming a new Diviſor.

First $r = 10,$
$+e\ 4,991838$
$a = 14,991838$ &c.

This Third Form is ſomething more troubleſome than the Two firſt; but a little Practice will render it very eaſy. And this is alſo a continued Series, (as before.)

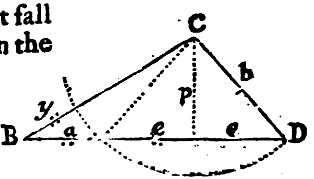
PROB. IV.

IN the Oblique Triangle *BCD*, there is given the Baſe (or Side) *BD*, with the Side *CD*; and if the Difference of the Sides *BC*, and *CD*, to wit, $BC - CD$, be multiplied into the Side *CD*, that Product muſt be equal to the Square of the Segment of the Baſe, *viz.* $= aa$.

By theſe to find out the Segment, *a*.

Let

Let	1	$BD = d$ } given { $d = 92$
	2	$CD = b$ } given { $b = 56$
And	3	$BC - DC = y$
Then	4	$by = aa$, by the Problem.



Suppoſe a Perpendicular, let fall from the Obtuſe Angle *C*, upon the the Side *BD*, call it *p*.

Alſo let $\frac{d-a}{2} = e$,

as in the Scheme.

Then

Prop. 1. {	5	$pp + ee = bb$
	6	$aa + 2ae + ee + pp = bb + 2by + yy$
$6 - 5$	7	$aa + 2ae = 2by + yy$
That is,	8	$a + 2e : 2b + y :: y : a$
		And this proves *Propoſition* 35, 36, 37. *Euc.* 3.
To wit,		That $BD : BC + CD :: BC - CD : a$.
8, ∵	9	$2by + yy = da$, for $d = a + 2e$
$4 \times \bar{2}$	10	$2by = 2aa$
$4 \circledcirc 2$	11	$bbyy = aaaa$
$11 \div bb$	12	$yy = \frac{aaaa}{bb}$.
$10 + 12$	13	$2by + yy = 2aa + \frac{aaaa}{bb} = da$
$13 \times bb$	14	$2bbaa + aaaa = bbda$
$14 \div a$	15	$aaa + 2bba = bbd$

In Numbers, $aaa + 6272a = 288512 = G$.

Let it be made $aaa + ba = G$.

Put	1	$r + e = a$
$1 \circledcirc 3$	2	$rrr + 3rre + 3ree = aaa$
$1 \times b$	3	$br + be = ba$
$2 + 3$	4	$r^3 + br + 3rre + be + 3ree = G$
$4 \div \bar{3}$	5	$\frac{1}{3}r^3 + \frac{1}{3}br + rre + \frac{1}{3}be + ree = \frac{1}{3}G$
$5 \div r$	6	$\frac{1}{3}rr + \frac{1}{3}b + re + \frac{\frac{1}{3}be}{r} + ee = \frac{\frac{1}{3}G}{r}$
$6 -$	7	$re + \frac{\frac{1}{3}be}{r} + ee = \frac{\frac{1}{3}G}{r} - \frac{1}{3}rr - \frac{1}{2}b = D$.

From hence ariſeth this *Theorem.* $\left\{ \frac{\mathfrak{D}}{r + \frac{\frac{1}{3}b}{r} + e} = e. \right.$

$6272 = b \qquad 288512 = G$

If $r = 40$, $rrr = 64000$, $br = 250880$.
But $250880 + 64000 = 314880$. *Ergo* $r < 40$.
Let $r = 30$

$\frac{1}{3}rr = 300 \qquad 96170,66 = \frac{1}{3}G$
$\frac{1}{3}b = 2090,6 \qquad 3205,688 = \frac{1}{3}G \div r$
$- 2390,6 = 2390,666 = \frac{1}{3}rr + \frac{1}{3}b$

$\frac{\frac{1}{3}b}{r} + r = 99,68$) $815,022 = \mathfrak{D}$
$+ e = 7 \qquad 742 \qquad (7 = e$
Diviſor $106, \qquad 73$

New $r = 37, \qquad 2599,207207 = \frac{1}{3}G \div r$
$\frac{1}{3}b \div r = 56,504 - 2546,999999 = \frac{1}{3}rr + \frac{1}{3}b$
$\frac{\frac{1}{3}b}{r} + r = 93,504 \qquad 52,207208 = \mathfrak{D}$
$+ e = ,5 \qquad 47,00 \qquad (,5 = e$
Diviſor $94,0 \qquad 5\,2072$
$+ e = ,05 \qquad 4\,7025 \qquad (5 = e$
Diviſor $= 94,05 \qquad 504708$
$+ e = ,005 \qquad 470295 \qquad (5 = e$
Diviſor $= 94,059 \qquad 344130$
$282177 \qquad (3 = e$

Laſt $r = 37,$
$+ e = ,5553$ $\Big\} = 37,5553 = a.$

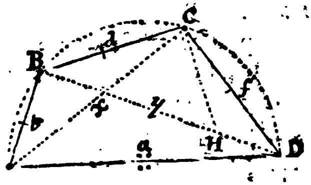
PROB. V.

THE three Chords (or Subtenſes) of three Arches, compleating a Semicircle, being each given: Thence to find the Diameter of that Circle.

Suppoſe $AB = 3 = b$, $BC = 4 = d$, and $\mathfrak{D}C = 5 = f$.

To find $A\mathfrak{D} = a$, the Diameter. Draw the two Diagonals, $B\mathfrak{D} = y$, and $ac = x$, Then is

$$\begin{array}{l|l|l} \textit{Prop.}1. & 1 & aa - ff = xx \\ & 2 & aa - bb = yy \\ \text{And} & 3. & da + bf = xy \end{array}$$



This Third Step I have often heard aſſerted, but never could ſee it clearly demonſtrated; which I undertake to do in this manner.

Make *CH* perpendicular to *BC*, then the Triangles *ACD*, and *BCH*, are alike, both Right-angled at *C*, and having the Angles at *A* and *B*, both meaſured by the Arch *CD*. Again, the Triangles *ABC* and *CHD* are alike; for the Angles at *A*, and *D*, ſtand both upon the Arch *BC*, and the Angles *DCH*, and *BCA*, are equal: For the Angles *BCA* — *ACH* = 90 Degrees, alſo the Angles *DCH* + *ACH* = 90 Degrees. That is, the Angles *DCH* + *ACH* = *BCA* + *ACH*; and conſequently the Angle *DCH* = *BCA*: Theſe being proved, it will be *AC* : *AB* :: *DC* : *DH*, and *AC* : *AD* :: *BC* : *BH*.

$$\begin{array}{l|l|l} \text{That is,} & 4 & x : b :: f : DH, \text{ and } x : a :: d : BH \\ 4 \because & 5 & \frac{bf}{x} = DH, \text{ and } \frac{da}{x} = BH, \text{ but } y = DH + BH \\ \text{therefore} & 6 & \frac{bf}{x} + \frac{da}{x} = y. \quad \text{Hence } bf + da = yx. \\ 1 \times 2 & 7 & aaaa - aaff - bbaa + ffbb = xxyy \\ 3 \ominus 2 & 8 & ddaa + 2dbfa + bbff = xxyy \\ 7,\ 8, & 9 & aaaa - ffaa - bbaa = ddaa + 2dbfa \\ 9 \div a & 10 & aaa - ffa - bba - dda = 2dbf \end{array}$$

Which in Numbers will be, $aaa - 50a = 120$.

Let it be made $aaa - ba = G$. This is an Equation of the Second Form of Cubicks, and may be ſolved with the ſame Theorem as the laſt, only changing the Sign of *b*.

Then it will be $re - \frac{\frac{1}{3}b}{r}e + ee = \frac{\frac{1}{3}G}{r} + \frac{1}{3}b - \frac{1}{3}rr = D$

From hence ariſeth this *Theorem* $\left\{ \frac{D}{r - \frac{\frac{1}{3}b}{r} + e} = e \right.$

M Let

Let $r = 6$, then $rrr = 216$, and $br = 300 > 216$, Ergo $r > 6$.

Suppoſe $r = 8$, $rrr = 512$, and $br = 400$, then $512 - 400 = 112$, hence it appears that $r > 8$, but not 9.

Let $r = 8$ $\qquad 120 = G \qquad b = 50$

$-\frac{1}{3}rr = 21{,}3333 \qquad 40 = \frac{1}{3}G$

$+\frac{1}{3}b = 16{,}6666 \qquad 5{,}0000 = \frac{1}{3}G \div r$

$-4{,}6667 \quad = \quad 4{,}6667 = \frac{1}{3}b - \frac{1}{3}rr$

$\frac{1}{3}b \div r = 2{,}083 \qquad {,}3333 = D$

$r - \frac{\frac{1}{3}b}{r} = 5{,}917 \qquad 2980 \qquad ({,}055 = e$

35300

$+e = {,}055 \qquad 29860$

1 Diviſor $5{,}96$

2 Diviſor $5{,}972$

New $r = 8{,}955$

$-\frac{1}{3}rr = 21{,}627675 \qquad 40 = \frac{1}{3}G$

$+\frac{1}{3}b = 16{,}6666666 \qquad 4{,}9658597144 = {}^{1}G \div r$

$-4{,}9610083 \quad = \quad 4{,}9610083333$

$r - \frac{\frac{1}{3}b}{r} = 5{,}9859 \qquad {,}0048513811 = D$

$478936 \qquad ({,}0008 = e$

$+e \quad {,}0008 \qquad 620211$

Diviſor $5{,}9867 \qquad 598671 \qquad (1 = e$

$+e \quad {,}00001 \qquad *2154000$

Diviſor $5{,}98671$ * $\qquad 1796013 \qquad (0359 = e$

* Here I deſiſt increaſing the Diviſor with e. $\qquad 3579980$

2993355

Laſt $r = 8{,}055 \qquad 5866250$

$+e = {,}000810359 \qquad 5389039$

$a = 8{,}055810359$

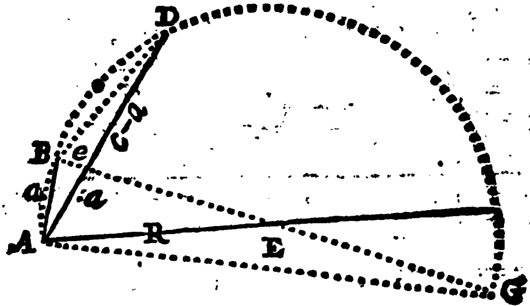
P R O B. VI.

THE Radius of a Circle, and the Chord (or Subtenſe) of an Arch being given, thence to find out the Chord of one Third Part of that Arch, which is the Triſecting of an Arch, or Angle.

Let $AE = R$, the Radius, and $AD = c$, the given Chord, (ſuppoſe of 60 Degrees.)

To find $AB=a$, (the Chord of 20 Degrees.)

Let $R=C=1,000000$, &c.



Prop.2.	1	$R:a::a:e$
	2	$c-a:e::2R-e:a\ (2R-e=BG-e)$
1 ∵	3	$\frac{aa}{R}=e.$
2 ∵	4	$ca-aa=2Re-ee$
3 ⊚ 2	5	$\frac{aaaa}{RR}=ee$
3 × 2R	6	$2aa=2Re$
6 — 5	7	$2aa-\frac{aaaa}{RR}=2Re-ee=ca-aa$
7 × RR	8	$2RRaa-aaaa=RRca-RRaa$
8 ÷ a	9	$2RRa-aaa=RRc-RRa$
9 +	10	$3RRa-aaa=RRc.$

This is the Third Form of Cubick Equations.

In Numbers, $3a-aaa=1$.

Let it be made $ba-aaa=G$.

Put	1	$r+e=a$
1 ⊚ 3	2	$r^3+3rre+3ree=aaa$
1 × b	3	$br+be=ba$
3 — 2	4	$br-r^3+be-3rre-3ree=G$
2 ÷ 3	5	$\frac{1}{3}br-\frac{1}{3}rrr+\frac{1}{3}be-rre-ree=\frac{1}{3}G$
5 ÷ r	6	$\frac{1}{3}b-\frac{1}{3}rr+\frac{\frac{1}{3}b}{r}e-re-ee=\frac{1}{3}G\div r$
6 ±	7	$\frac{\frac{1}{3}b}{r}e-re-ee=\frac{\frac{1}{3}G}{r}+\frac{1}{3}rr-\frac{1}{3}b=\mathfrak{D}$

From hence ariſeth this *Theorem* $\left\{ \dfrac{D}{\dfrac{\frac{1}{3}b}{r} - r - e} = e \right.$

Being $c = 1$. $r > ,3$ but $r < ,4$

Let $r = ,3$ $1 = G$ $b = 3$

$\frac{1}{3}rr = ,03$ $,333333 = \frac{1}{3}G$

$-\frac{1}{3}b = 1,00$ $1,111111 = \frac{1}{3}G \div r$

$- ,97 =$ $,97 = \frac{1}{3}rr - \frac{1}{3}b$

$\frac{1}{3}b \div r = 3,3333$ $,141111 = D$ $(,04 = e$

$-r = ,3$ 119732

$\frac{\frac{1}{3}b}{r} - r = 3,0333$ 213790 $(7 = e$

209041

$-e$ $,047$

Diviſor $2,9933$

Diviſor $2,9863$

New $r = ,347$ $,960614793458 = \frac{1}{3}G \div r$.

$\frac{1}{3}rr - \frac{1}{3}b = -$ $,959863666667$

$\frac{\frac{1}{3}b}{r} - r = 2,534844)$ $,000751126791 = D$

$-e$ $,0002$ 50692 $(,0002 = e$

Diviſor $2,5346$ 2442067 $(9 = e$

$-e$ $,00009$ 2281695

Diviſor $2,53455$ 1609791 $(6 = e$

$-e$ $,000006$ 1520728

Diviſor $2,534548$ 8900036 $(35 = e$

Laſt $r = ,347$ 7603644

$+e$ $,00029635$ 1296386

$a = ,34729635$ 1267479

In the preceding Examples, I have inſerted ſuch Problems, as would produce the three Forms (as they are uſually called) of Square, and Cubick Adfected Equations, (each in its Order;) that is, ſuch as conſiſt of the higheſt Power thereof, mixt with its Root.

How other Equations that are Mixt, or Adfected with their intermediate Terms, (or Inferiour Powers) to wit, $aaa + baa - ca = d$, or $baa - ca - aaa = d$, &c.

may be reduced to ſome of the three foregoing Forms, might here be ſhewed by thoſe commonly called *Cardan's* Rules, or otherwiſe; but ſuch Proceſs is too large and tedious to be inſerted in this ſmall Tract: Neither (in truth) have we any need of ſuch Expedients, as the caſting off the ſecond Term of an Equation, *&c.* which was invented purely to render the ſame more fit and eaſy for a Solution in Numbers. For by the Methods of converging Series, any Equation may be as well ſolved, (ſtill keeping its own Form, as the Problem produced it) as thoſe of the three Forms.

I have alſo omitted the Doctrine of *Surds* for the like reaſon, to wit, becauſe it requires a large Explanation to render it intelligible, without which, it's very intricate and troubleſome, and rather puzzles or confounds a Learner's Genius, than improves it; neither could I ever (yet) perceive any great uſe or advantage thereby, in reſolving any Problem, (and afterwards we have now no occaſion for it.) For admit a Problem, or any part thereof, were propoſed in *Surds*, it's much eaſier (in my opinion) to bring it out of ſuch *Surds*, before they be engaged in any Proceſs, than after they are become intangled, and mixed with other Quantities concern'd therein. I could likewiſe have inſerted Rules for diſcovering of the ſeveral Roots of an affected Equation, whether affirmative or negative, *&c.*

How ſuch Work may be performed, is already ſhewed.

Notwithſtanding that, I have in this ſecond Edition added a Chapter in its proper place, which ſhews the young Algebraiſt how to manage ſuch *Surd Quantities* as may happen (and often do ariſe) in the Solution of hard or difficult Queſtions.

But I content myſelf in this Tract with one Root only, that is, ſuch a one as ſuits or agrees beſt with the Nature and Deſign of the Problem propoſed, leaving thoſe Speculations to be inquired after (by ſuch as deſire them) in the Works of *Des Cartes*, or Dr. *Wallis*'s Treatiſe of *Algebra*, wherein they may meet with ample ſatisfaction in each particular Caſe, *&c.*

Before I proceed to the reſolving of other kinds of affected Equations, it may be convenient to ſay ſomething of thoſe that are already done.

Perhaps it may be ſuggeſted by ſome, that this Method requires too many Operations in Diviſion, as two, three, or ſometimes more, according to the Number of Terms in the Equation.

That it doth require ſeveral Diviſions, is true; but withall conſider, that thoſe Diviſions are performed with ſmall Diviſors, and that each Operation is in order to bring down both the Reſolvend and Diviſor into lower Terms, which was the very thing I firſt aimed at.

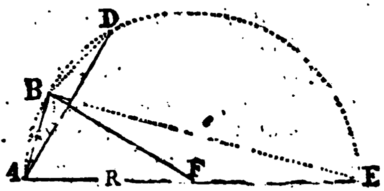
However, to accommodate ſuch as prefer *Multiplication* (tho' the Factors be large) before *Diviſion*, (notwithſtanding the Diviſors be ſmall) I will here inſert a different Method of ordering the ſeveral Powers or the Root *r*, with the Coefficients, &c. which perhaps may ſeem eaſier to ſome, and therefore may pleaſe better; tho' both come to the ſame in the concluſion.

And ſince I have entered upon the Subject of angular Sections, I will proceed a little further therein; and reſolve ſome of the odd Sections, *viz.* the fifth, ſeventh, and ninth, paſſing over thoſe of the even Powers, to wit, the fourth, ſixth, and eighth, becauſe they may be reſolved by that of the Biſection only; for that reaſon it will be convenient (tho' out of its due courſe) to inſert the Biſection of an Angle.

PROB. VII.

THE Radius of a Circle, and the Chord of an Arch, being each given, to find the Chord of half that Arch, which is to biſect an Arch, or Angle.

Let | 1 | $AF = R$ the Radius $= AD$, Chord of 60 Deg.
| 2 | $AD = c$
| 3 | $BA = a$



$R = 1$,
$C = 1$,
Let $BE = e$
Then

Prop. 1. | 4 | $4RR - aa = ee$
Prop. 2. | 5 | $a : c :: R : e$, therefore it is
5, | 7 | $aa : cc :: RR : ee$
That is, | 8 | $aa : cc :: RR : 4RR - aa = ee$
7 | 8 | $4RRaa - aaaa = RRcc$.

Which in Numbers is, $4aa - aaaa = 1$

Let $r + e = a$, as before,

Then will $- rrrr - 4rrre - 6rree = - aaaa$

And $\underline{+ 4rr + 8re + 4ee = + 4aa}$.

By the *Prob.* $r > {,}5$ but $r < {,}6$ therefore

Let $r = {,}5$ then $rr = {,}25$ and $rrr = {,}125$

and $rrrr = {,}0625$ then it will be

$$\left.\begin{array}{l} - {,}0625 - {,}5e - 1{,}5ee = - aaaa \\ \underline{+ 1{,}0000 + 4{,}0e + 4{,}0ee = 4aa} \end{array}\right\} = 1$$

That is, $+ {,}9375 + 3{,}5e + 2{,}5ee = 1$

And $3{,}5re + 2{,}5ee = {,}0625$

Then will $1{,}4e + ee = {,}025 = D$

$$\textit{Conſequently}, \left\{ \frac{D}{1{,}4 + e} = e \right.$$

$1{,}4$ $\qquad\qquad$ $\dot{,}02\dot{5}0 = D$

$+ e = \underline{{,}0176}$ $\qquad\qquad$ $\underline{141}$ $\qquad$ $({,}0176 = e$

Divisor $1{,}4176$ $\qquad\qquad$ 10900

$\underline{9919}$

First $r = {,}5$ $\qquad\qquad$ 98100

$+ e = \underline{{,}0176}$ $\qquad\qquad$ $\underline{85056}$

New $r = {,}5176$

I take but $,517 = r$, which being involved as before, the Numbers will be

$$\begin{array}{l} - {,}0714434 - {,}55275e - 1{,}6037ee \\ \underline{+ 1{,}0691560 + 4{,}13600e + 4{,}0000ee} \end{array}$$

That is, $,9977126 - 3{,}58425e + 2{,}3963ee = 1$

Then $3{,}58425e + 2{,}3963ee = {,}0022874$

And $1{,}49593e + ee = {,}0009547120 = D$

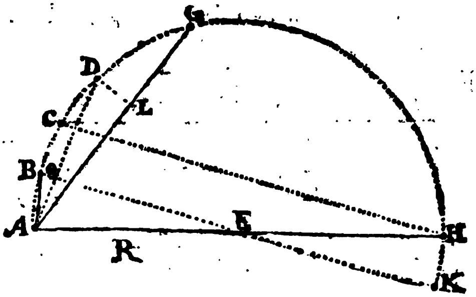
$$\textit{Conſequently}, \left\{ \frac{D}{1{,}49593 + e} = e \right.$$

1,49593	,0009547120 = D	
+ e = ,0006	89790	(,0006 = e
Diviſor 1,4965	568120	
+ e = ,000037	448968	(37 = e
Diviſor 1,496567 *	11915200	
	10475969	
* Here I deſiſt increaſing the Diviſor.	*14392310	(9616 = e
	13469103	
	923207	
	897936	
Laſt r = ,517	25271	
+ e = ,0006379616	14965	
a = ,5176379616	10306	
	&c.	

PROB. VIII.

THE Radius of a Circle, and the Chord of an Arch (ſuppoſe of 60 *gr.* as before) being given, to find the Chord of one fifth part of that Arch (to wit, of 12 *gr.*)

Let $AE = R$, Radius $\Big\}$ given $\Big\{ R = 1$
and $AG = c$, Chord, $\Big\}$ given $\Big\{ c = 1$
$AB = a$, the Chord ſought.



By the third Step in the Proceſs of the Triſection of an Angle, *pag.* 83, it was found that $\frac{aa}{R} = e$; and by the tenth Step, $\frac{3RRa - aaa}{RR} = AD$

Make DL perpendicular to AD.
Then are the Triangles AHC, and DAL, alike.
And $AH : CH :: AD : AL$. But $AH = 2R$.

But $CH = BK - e$, that is, $2R - \frac{aa}{R}$

Again, $AD = \frac{3RRa - aaa}{RR}$, from hence it will be

$$2R : 2R - \frac{aa}{R} :: \frac{3RRa - aaa}{RR} : AL$$

That is, $\frac{6R^4a - 5RRaaa + a^5}{2RRRR} = AL$.

But $AL = \frac{1}{2}AG + \frac{1}{2}CD$, and $\frac{1}{2}CD = \frac{1}{2}a$

therefore, $\left\{\frac{6R^4a - 5RRaaa + aaaaa}{2RRRR} = \frac{1}{2}c + \frac{1}{2}a\right.$

That is, $aaaaa - 5RRaaa + 5RRRRa = RRRRc$

In Numbers $aaaaa - 5aaa + 5a = 1$

Put $r + e = a$

$$\text{Then}\left\{\begin{array}{l} + rrrrr + 5rrrre + 10rrree = aaaaa \\ - 5rrr - 15rre - 15ree = -5aaa \\ + 5r + 5e \ldots\ldots\ldots = 5a \end{array}\right.$$

If $c = 1$. $r > ,2$ but $r < ,3$ by the Scheme,

$$\text{Then}\left\{\begin{array}{l} + ,00032 + ,008e + ,08ee = aaaaa \\ - ,04 - ,600e - 3,00ee = -5aaa \\ + 1,0 + 5,0e \ldots\ldots\ldots = 5a \end{array}\right.$$

That is, $+ ,96032 + 4,408e - 2,92ee = 1$
Then $4,408e - 2,92ee = ,03968$
And $1,5e - ee = ,01356763 = D$

Conſequently, $\left\{\frac{D}{1,5 - e} = e\right.$

$$\begin{array}{lrl} & 1,5 & \\ -e = & \underline{,009} & \\ \text{Diviſor} & 1,491 & \end{array} \qquad \begin{array}{r} \dot{,}0\dot{1}3\dot{5}6\dot{7}63 = D. \\ \underline{13419} \\ 14863 \end{array} \qquad (,009 = e$$

Firſt $r = ,2$
$+ e = \underline{,009}$
New $r = 2,09$ which being ordered as before,

$$\text{Then}\left\{\begin{array}{llll} +\,{,}0603987782 & +\,{,}00954e & +\,{,}09129ee \\ -\,{,}0456466450 & -\,{,}65521e & -\,3{,}13500ee \\ \underline{+\,1{,}045} & \underline{+\,5{,}0e} & \end{array}\right.$$

That is, $,9997521332 + 4,35433e - 3,0437067ee$

Then $4,35433e - 3,0437ee = ,0002478668$

Each part divided by 3,0437 then it will become

$$1{,}430604e - ee = {,}00008143601 5 = \mathcal{D}$$

$$\textit{Conſequently,}\left\{\frac{\mathcal{D}}{1{,}430604 - e} = e\right.$$

$$\begin{array}{lrl}
1{,}430604) & {,}00008143601 5 = \mathcal{D} & \\
-\,e = \underline{{,}00005} & \underline{715275} & ({,}00005 = e \\
\text{Diviſor } 1{,}43055 & 9908515 & (6 = e \\
-\,e = \underline{{,}000006} & \underline{8583288} & \\
\text{Diviſor } 1{,}430548 & 132522700 & (9 = e \\
-\,e = \underline{{,}0000009} & \underline{128749239} & \\
\text{Diviſor } 1{,}4305471 & 37734610 & (263 = e \\
 & \underline{28610942} & \\
 & 9123668 & \\
 & \underline{8583282} & \\
\text{Laſt } r = {,}209 & 540386 & \\
+\,e = \underline{{,}0000569263} & \underline{429162} & \\
a = {,}2090569263 & \&c. &
\end{array}$$

I have here (according to Promiſe, *pag.* 86.) preſented you with another Method of Reſolving Equations, very different from the firſt; for in that the Members of the Solution, (to wit, $r + e$) together with the Coefficients, are ſo ordered, that from them (ſtill remaining in their Species) there ariſeth a Theorem for the Solution of the Equation.

But in this Method, it is otherwiſe; for when the Members $r + e$, are involved according to the Equation, and multiplied into their reſpective Coefficients, then are they to be brought into Numbers, and a due Collection made thereof, according to their Signs and Places.

Next, the abſolute Numbers therein (unmix'd with e) are to be compared with the Reſolvend, and added to, or ſubſtracted therefrom, as their Sign denotes; after which, each part is to be divided by the Factor found, prefixed to *ee*, that ſo the ſame may be cleared: Conſequently, a Diviſor will ariſe, which muſt be either increaſed or leſſened by the Converging *e*, according to its Sign; the which, *I preſume*, doth plainly appear, in the Solution of this and the laſt Equation.

I ſhall give an Example or two more, that ſo the young Algebraiſt may the better judge of the difference betwixt this and the other, and practiſe which he fancies.

Note, That this Mehtod doth triple the Number of Places in the Root, at each Operation, as the other doth.

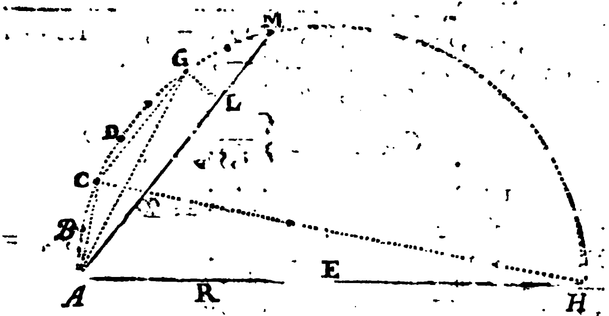
P R O B. IX.

THE Radius of a Circle, and the Chord of an Arch being given, to find the Chord of the Seventh part of that Arch.

Let $AE = R$ Radius, $\Big\}$ given $\Big\{$ $R = 1$

and $AM = c$ Chord, $\Big\}$ given $\Big\{$ $c = 1$, the Chord of 60

Then $AB = a$, the Chord of 8 Degr, 34′ 19″.



By the laſt Problem, it was found that

$$AG = \frac{aaaaa - 5RRaaa + 5RRRRa}{RRRR}$$

And by that in *pag.* 83. it was found that

$$\frac{3RRa - aaa}{RR} = AD. \quad \text{And } CH = \frac{2RR - aa}{R}$$

Make GL perpendicular to AM, then are the Triangles AHC and GAL alike; therefore it will be

$$AH : CH :: AG : AL. \quad \text{But } AH = 2R$$

And $AL = \frac{1}{2}AM + \frac{1}{2}CG$, but $\frac{1}{2}CG = \frac{1}{2}AD$

then

then by taking thoſe that are equal, it will be

$$2R : \frac{2RR - aa}{R} :: \frac{a^5 - 5RRa^3 + 5R^4a}{RRRR} \tfrac{1}{2}c + \frac{3RRa - aaa}{2RR}$$

If $R = 1$, as before, then it will become

$$2 : 2 - aa :: a^5 - 5aaa + 5a : \tfrac{1}{2}c + \frac{3a - aaa}{2}$$

That is, $-a^7 + 7a^5 - 14aaa + 7a = 1$

Put $r + e = a$. Then it will be

$$\left.\begin{array}{l} -r^7 - 7r^6e - 21r^5ee = -a^7 \\ +7r^5 + 35r^4e + 70r^3ee = +7a^5 \\ -14r^3 - 42rre - 42ree = -14a^3 \\ +7r + 7e \quad = +7a \end{array}\right\} = c$$

If $c = 1$ then $r > ,1$ but $r < ,2$ by the Scheme.

$$\text{Then} \left\{\begin{array}{lll} -,0000001 & -,0000007e & -,00021ee \\ +,00007 & +,00035e & +,07ee \\ -,014 & -,42e & -4,2ee \\ +,7 & +7,0e & \end{array}\right.$$

That is, $+,6860699 + 6,583493e - 4,130210ee = 1$

Then $6,583493e - 4,130210ee = ,3139301$

And $1,593e - ee = ,076008 = D$

$$\textit{Conſequently} \left\{ \frac{D}{1,539 - e} = e \right.$$

1,593		
$-e =$,04	,076008 $= D$	
Diviſor 1,55	620	(,049 $= e$
$-e =$,009	14008	
Diviſor 1,544	13896	

Laſt $r = ,1$ to which add ,049 New $r = ,149$

$$\begin{array}{lll} -,00000163044 & -,0000766e & -,001542ee \\ +,00051407839 & +,0172508e & +,231566ee \\ -,04631128 6 & -,932442e & -6,258ee \\ +1,043 & +7,0e & \end{array}$$

$+,99720116195 + 6,0847322e - 6,027976ee = 1$

That is, $1,0094137e - ee = ,000464340511 = D$

1,0094137

1,0094137 ,00046434051 1 $= D$

$-e =$,0004602 40360 (,0004 $= e$

Divisor 1,0090 607405 (6 $= e$

Divisor 1,00895 605370

Divisor 1,0089535 2035100 (02 $= e$

Here I deſiſt forming 20179070

a new Diviſor. 1720700 (01 $= e$

1008953

Laſt $r =$,149 } ,149460201 $= a$
$+e =$,000460201 }

PROB. X.

THE Radius of a Circle, and Chord of an Arch being given, to find the Chord of one ninth part of that Arch.

This Problem needs no Scheme, but a Conſideration of the Equation ariſing from *Prob.* VI. *viz.* Of the Triſection of an Angle: There it was found that

$$3RRa - aaa = RRc.$$

And if it be made $R = c = 1$ in this Problem, as it was in that, then it will be

$$3a - aaa = 1$$

And here we are to find a third part of that Arch whereof a is the Chord, for $\frac{1}{3}$ of $\frac{1}{3} = \frac{1}{9}$ In order thereunto, let it be made $3y - yyy = c$

And for the Chord ſought, put a; then will

$$3a - aaa = y, \text{ and } 9a - 3aaa = 3y$$

Alſo, $27aaa - 27aaaaa + 9a^7 - a^9 = yyy.$

And from theſe will ariſe this Equation;

$$a^9 - 9a^7 + 27a^5 - 30a^3 + 9a = 3y - yyy = c$$

If as before, $c = 1$ the Chord of 60 Degrees, then will a be the Chord of 6 Degrees, 40 Minutes,

Put $r + e = a$, then there will ariſe theſe, *viz.*

$$+r^9 + 9r^8e + 36r^7ee = +a^9$$
$$-9r^7 - 63r^6e - 189r^5ee = -9a^7$$
$$+27r^5 + 135r^4e + 270r^3ee = 27a^5$$
$$-30r^3 - 90r^2e - 90re = -3a^3$$
$$+9r + 9e \ldots\ldots\ldots = 9a$$

$c = 1$

$c = 1$ then $r > ,1$ but $r < ,2$ becauſe we are to find the ninth part of the Arch : Let $r = 1$, which being ordered as the Species directs, will produce theſe Numbers.

$$\begin{array}{lll}
+\,,000000001 & +\,,00000009e & +\,,0000036ee \\
-\,,0000909 & -\,,000063e & -\,,00189ee \\
+\,,00027 & +\,,0135e & +\,,27ee \\
-\,,03 & -\,,9e & -\,9,0ee \\
+\,,9 & +\,9,0e &
\end{array}$$

That is $,870269101 + 8,113437o9e - 8,7318854ee = 1$

Then $8,113437o9e - 8,7318864ee = ,129730899$

And $,92916e - ee = ,014857 = \mathcal{D}$

$$\text{Conſequently,}\left\{\frac{\mathcal{D}}{,929 - e} = e\right.$$

$$\begin{array}{lll}
 & ,92916 & ,014857 = \mathcal{D} \\
-e = & \underline{,0163} & \underline{91} \qquad (,0163 = e \\
\text{Diviſor} & \underline{,91} & 5757 \\
\text{Diviſor} & \underline{,913} & \underline{5478} \\
\text{Diviſor} & ,9128 & 27990 \\
 & & 27384
\end{array}$$

Laſt $r = ,1 + ,0163 = ,1163 = r$ I take $r = 116$

Which being ordered as before, will be

$$\begin{array}{lll}
+\,,0000000003829 & +\,,000000295e & +\,,00001017ee \\
-\,,000002543598 & -\,,000153493e & -\,,0039642ee \\
+\,,000567092247 & +\,,0244436 26e & +\,,4214719ee \\
-\,,04682688 & -\,1,21104e & -\,10,44ee \\
+\,1,044 & +\,9,0e &
\end{array}$$

$+\,,99773767147 8 + 7,81325042 8e - 10,9225433ee = 1$

That is, $7,81325e - 10,0225433ee = ,002262328522$

And then, $7795676e - ee = ,0022572 3995 = \mathcal{D}$

$$\text{Conſequently}\left\{\frac{\mathcal{D}}{,7795676 - e} = e\right.$$

,7795676)

;7795676)	;000225723995 $=$ D	
$-e=$,0002896	15586	(,0002 $= e$
Diviſor ;7793	698639	(8 $= e$
Diviſor ,77928	623424	
Diviſor ,7792780	7521595	(9 $= e$
	7013502	
The Diviſor remains	50809300	(652 $= e$
without new forming.	46756680	
Laſt $r=$,116	40526200	
$+e=$,000289652	38963900	
$a=$,116289652	15623000	
	15585560	

I have choſen to proſecute theſe Angular Sections to this height (*which is further than I ever ſaw*) for that I was willing to ſhew by Example, with what eaſe and expedition this Method reſolveth high Adfected Equations; the Root converging as quick in them, as it doth in thoſe of a lower Rank.

Alſo becauſe from hence may be propoſed a much ſpeedier and eaſier Way of Approximation, towards the finding out the Circumference of a Circle, than what the Antients had; for *Archimedes* (*and others ſince him*) proceeded therein, by the continual Biſection of an Arch: as at *Prob.* 7. beginning with the Chord (or Subtenſe) of 60 Degrees; that is, of $\frac{1}{6}$ part of the Circumference, and from thence found the Chord of $\frac{1}{12}$, then of $\frac{1}{24}$, then of $\frac{1}{48}$, then of $\frac{1}{96}$; then of $\frac{1}{192}$, then of $\frac{1}{384}$, and ſo on, according to the nearneſs they intended to bring the Chord to the Arch itſelf. And this way of proceeding (*as they reſolved that Equation*) requires Twenty-ſeven ſeveral Extractions of the Square Root (that is, *Root out of Root*) to obtain the Chord of $\frac{1}{384}$, part of the Circumference. And no doubt but they made uſe of thoſe numerous Extractions, for want of other Expedients to facilitate the Work; otherwiſe (*it is but reaſonable to ſuppoſe*) they would not have ſpent ſo much Time and Labour, as muſt neceſſarily be required in the performance thereof; both which are by this Method much abbreviated, and not only ſo, but may be more accurately performed, becauſe here is not required the Extraction of a Surd Root, out of a Surd Root, ſo often repeated: for by the Reſolving of one Equation, here is produced the Chord of $\frac{1}{54}$ part of the Circumference, beginning at the Chord $\frac{1}{6}$; for $\frac{1}{9}$ of $\frac{1}{6}=\frac{1}{54}$; and by a Solution of the ſame Equation a ſecond time, it will produce the Chord $\frac{1}{486}$ part of the Circumference; for $\frac{1}{9}$ of $\frac{1}{54}=\frac{1}{486}$. And if further Exactneſs be required, it is but repeating it once more, and then it will produce the Chord of $\frac{1}{4374}$, part of the Circumference. *Which is nearer than can be obtained by the repetition of fifty-three ſeveral Extractions, by the Method of Biſection before mentioned.*

And the better to ſhew how near two Solutions will approach to the (common received) Circumference of a Circle, whoſe Diameter is 2, I have inſerted a ſecond Proceſs, taking the ſame Æquation Equal to the Chord laſt found, to wit,

$$a^9-9a^7+27a^5-30a^3+9a=,116289652$$

And for the ſame Reaſon that the firſt $r=,1$ in the laſt Proceſs, in this it muſt be made $r=,01$.

And here it is to be noted, that becauſe $r=,01$ then will r^9-9r^7 the two firſt Members of the Solution, have one 17, and the other 13 Cyphers before their ſignificant Figures; both which being of ſo ſmall value, may be rejected; by which the Solution will become contracted to this following.

$$\left.\begin{array}{l} +27r^5+135r^4e+270r^3ee \\ -30r^3-90r^2e-90ree \\ +9r+9e \end{array}\right\}=,116289652$$

$$\text{Then}\left\{\begin{array}{lll} +,0000000027 & +,00000135e & +,00027ee \\ -,00003 & -,009e & -,9ee \\ +,09 & +9,000e & \end{array}\right.$$

Viz. $,0899700027+8,99100135e-,89973ee=,116289652$

Then $8,99100135e-,89973ee=,0296316493$

And $9,993e-ee=,0292528306=\mathcal{D}$

$$\text{Conſequently}\left\{\frac{\mathcal{D}}{9,993-e}=e\right.$$

9,993)

$9{,}993)\ {,}0292528306 = \mathfrak{D}\ ({,}002928 = e$

$-e = {,}002 \qquad 19982$

Divisor $9{,}991 \qquad 927083$

$-e = {,}0009 \qquad 899109$

Divisor $9{,}9901 \qquad 2797406$

$-e = {,}00002 \qquad 1998016$

Divisor $9{,}99008 \qquad 7993900$

7992064

$$\left.\begin{array}{l}\text{Laſt } r = {,}01 \\ +e = {,}002928\end{array}\right\} = {,}012928 = a$$

Or rather new $r = 0{,}012928$ for a ſecond Operation; and let $r + e = a$, as before.

Then the ſeveral Powers of $r + e$ being raiſed, and multiplied into their reſpective Coefficients: as in the laſt Work, ſave only now all the Powers wherein ee is concern'd may be wholly rejected, and then the Value of e may be found by plain Diviſion. The Numbers ſo produced will be theſe:

$$\mathit{Viz.}\left\{\begin{array}{l} +{,}0000000097 + {,}000003e \\ -{,}0000652086 - {,}015041e \\ +{,}116352 \ldots + 9{,}000000e \\ \hline +{,}1162868011 + 8{,}984962e = 0{,}116289652 \end{array}\right.$$

Hence it will be $8{,}98496e = 0{,}000002851 = \mathfrak{D}$

$$\mathit{Conſequently}\left\{\frac{\mathfrak{D}}{8{,}98496} = e\right.$$

That is, $8{,}98496)\ {,}000002851000\ ({,}0000003173 = e$

2695488

1555120

898496

6566240

6289472

2767680

2695488

&c.

Laſt $r = 0{,}012928$

$+e = 0{,}0000003173$

$r + e = 0{,}0129283173 = a$

the Chord of $\frac{1}{486}$ part of the Circle's Periphery.

Conſequently 0,0129283173 × 486 = 6,2831625, &c. will be the Circle's Periphery produced from this Solution; being ſomething leſs than 6,2831853, which is the Periphery very near the truth.

There are other Methods of approaching to the Circle's Periphery; (*for exactly true, it cannot be found*:) but this way by the Chord of an Arch, (*be it perform'd by what Section you pleaſe*) I take to be the moſt Natural and Demonſtrable of all others.

Now by the help of this Periphery 6,2831625 (or rather 6,2831853) whoſe Diameter is 2, not only the Periphery of all other Circles and their Area's, but alſo the Solidity and Superficies of any Sphere or Globe may be eaſily found.

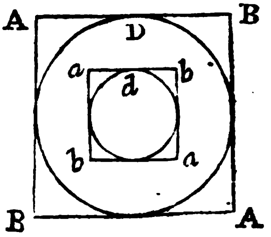
PROP. XI.

Of finding any Circle's Periphery *and* Area.

CIRCLES Peripheries may be found by the following

Proportion, { *As the Diameter of any Circle* : *Is in proportion to its Periphery* : : *So is any other Diameter* : *To its Circle's Periphery.*

This Proportion may not only be Illuſtrated, *but alſo Demonſtrated*, by the annexed Figure.



Wherein ſuppoſe AB = the Side of the greateſt Square; and D = AB, the Diameter of its inſcribed Circle; and P = its Periphery. Let D = 2, then will $p = 6,2831853$, as above.

Again, Suppoſe ab = the Side of the leſſer Square; and $d = ab$, the Diameter of its inſcribed Circle; and let p = its Periphery.

Then I think it's eaſy to conceive, by the Figure, That D : P : : d : p, *viz.* 2 : 6,2831853 : : 1 : 3,1415926 = p the Periphery of that Circle whoſe Diameter is $d = 1$

Which gives this general *Theorem* :

viz. { *The Peripheries of Circles are in proportion one to another, as their Diameters are.*

Conſequently, if the Diameter of any propoſed Circle be multiplied with 3,1415926; or as in common Practice, with 3,1416; the Product will be the Periphery required.

For to find the Circle's Area; 'tis very obvious, only by Inſpection of the Figure,

That { *As the Perimeter of the Square* (viz. *the Sum of its four Sides*) *: Is in proportion to its Area :: So is the Periphery of its inſcribed Circle : To its Area.*

Thus if D = the Side of the Square, and Diameter of its inſcribed Circle: Then 4 D = the Perimeter, DD = the Area of the Square, and 3.1416 D = *p* the Periphery of its inſcribed Circle, ſound as before.

And then it will be, 4D : DD :: *p* : to the Circle's Area.

Or 4*d* : *dd* :: *p* : to its Circle's Area.

If *d* = 1, then 4*d* = 4. *dd* = 1 × 1 = 1. and *p* = 3.1416*d*, as before.

Then 4 : 1 :: 3.1415 : 0,7854 the Circle's Area, whoſe Diameter is 1.

Now this way of finding the Area of a Circle, by comparing it with that of the Square, not only agrees with, but alſo proves the ſecond Propoſition of the twelfth Book of *Euclid*'s Elements.

Propoſition, { *The Area's of all Circles, are in proportion one to another; as the Squares of their Diameters are.*

For if D = the Diameter of one Circle, and *d* = the Diameter of another Circle; then 0,7854 DD = the Area of that Circle; and 0,7854*dd* will be the Area of the other Circle.

But 0,7854DD : 0,7854*dd* :: DD : *dd*. Ergo, &c.

From hence it will be eaſy to deduce the two following Rules:

viz. That if the Diameter of any propoſed Circle be multiplied with 3,1416; the Product will be the Periphery of that Circle: And if the Square of any Circle's Diameter be multiplied with 0,7854; that Product will be the Area of that Circle, in ſquare Meaſures (*or Parts*) of the ſame name with thoſe of the Diameter: *viz.* if the Diameter be Inches, then will the Area be ſquare Inches; and ſo for other Meaſures.

Now what is here done, as to the Proportions betwixt the Square and its inſcribed Circle, may alſo be applied to the Cube, and its inſcribed Sphere or Globe.

PROB. XII.

To find the Solidity and Superficies of any Sphere, &c.

THUS if we ſuppoſe $d = 1$ to be the Side of the Cube, and Diameter or Axis of its inſcribed Sphere; then will $6d$ be the Perimeter or Sum of all the Sides; ddd its ſolid Content; and $3{,}1416d = p$, the Periphery of the Sphere's greateſt Circle.

And the Proportion will be,

$$6d : 3{,}1416d :: ddd : \frac{3{,}1416dddd}{6d} = 0{,}5236ddd,$$ the ſolid Content of the inſcribed Sphere, whoſe Diameter or Axis is 1.

Whence it follows, that becauſe $d = 1$, whoſe Cube is $1 \times 1 \times 1 = 1$; therefore if the Cube of the Axis of any propoſed Sphere, be multiplied into or with 0,5236, the Product will be the ſolid Content of that Sphere.

That is, Suppoſing $D =$ the Diameter or Axis of any Sphere; then $0{,}5236DDD$ will be the ſolid Content of that Sphere.

For $ddd : DDD :: 0{,}5236ddd : 0{,}5236DDD$; which plainly proves the Truth of that difficult Propoſition, 18 *Euclid* 12.

viz. { *That Spheres are in proportion one to another;*
As the Cubes of their Diameters are.

Having thus obtained a Rule for finding the ſolid Content of any Sphere, by comparing it to its circumſcribing Cube, &c. let us ſee how to find the ſuperficial Content, by help of the Cube: Which, I preſume, may be done by this following

Proportion, { *As the ſolid Content of any Cube : Is to its Superficies : : So is the ſolid Content of its inſcribed Sphere : To its Superficies.*

viz. Suppoſe $D =$ the Side of the Cube and Axis of the Sphere, as before.

Then $6DD =$ the Superficies of that Cube; DDD its Solidity; and $0{,}5236DDD$ the ſolid Content of the inſcribed Sphere: Then it will be,

$$DDD : 6DD :: 0{,}5236DDD : \frac{0{,}5236DDD \times 6DD}{DDD}$$

$= 3{,}1416DD$, the Superficies of the Sphere.

This Equation gives the following Theorem:

Theorem: *The Superficies of every Sphere (or Globe) is equal to four times the Area of its greateſt Circle.*

That is, if $D =$ the Axis of any Sphere, then will $0{,}7854DD$ be the Area or Superficies of its greateſt Circle; and $0{,}7854DD \times 4 = 3{,}1416DD$ the Superficies of the whole Sphere, as above.

Therefore it will always be,

As $1 : 3{,}1416 :: DD : 3.1416DD$ the Superficies of any propoſed Sphere.

Thus have I preſented the young Algebraiſt with a full Account of the Circle, and Sphere, &c. not only of the bare Rules by which they may be found; but have traced their Proportions from their original Foundations, or firſt Principles; and dare preſume to ſay, they are rendred ſo plain, clear, and eaſy to be underſtood, as he will not meet with the like in any other Treatiſe that was ever yet publiſhed.

PROB. XIII.

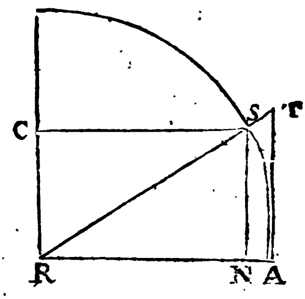
I Shall now proceed to apply the aforeſaid Periphery 6,283185 of the Circle, whoſe Radius is Unity or 1, to the making of Natural Sines and Tangents, by two ſeveral Methods. And in order to perform either of them, it muſt be preſumed, that the natural Sine of one Minute doth ſo inſenſibly differ from the Length of the Arch of one Minute, that it may be taken for the ſame.

Conſequently, *As the Periphery in Minutes : Is to the Periphery in equal Parts of the Radius :: So is one Minute : To the Parts agreeing to that Minute.*

That is, $21600' : 6{,}283185 :: 1' : 0{,}000290888 =$ the natural Sine of one Minute; which agrees with the largeſt Table of Sines I ever ſaw.

Having thus got the Sine of one Minute, its Co-Sine may be thus found:

Suppoſe $RA = RS$ the Radius of any Circle, $SN =$ the Sine of the Arch SA. Then $RN = CS$, is the Co-Sine of that Arch. But $\square RS - \square SN = \square RN$, conſequently $\sqrt{\square RS - \square SN} = RN$.



That is, From the Square of the Radius, ſubſtract the Square of the Sine of 1′, the Square Root of the Remainder will be the Co-Sine of 1′, per *Chap.* 9. *Prop.* 1. In Numbers, the Sine of 1′ is 000290885, its Square is 0,000000084612; and 1 — 0,000000084612 = 0,9999999153 88, the Square Root thereof is ,99999995 = the Co-Sine required.

The Sine and Co-Sine of one Minute being thus obtain'd, all the reſt of the Sines in the Quadrant may be gradually calculated by Mr. *Michael Dary*'s Sinical Proportions; which I ſhall here inſert, to the ſame effect as they are in his Miſcellanies; and then explain and demonſtrate the Truth of thoſe Proportions.

If a Rank of Arches be equi-different;

Then { *As the Sine of any Arch in that Rank : Is to the Sum of the Sines of any two Arches equally remote from it on each Side :: So is the Sine of any other Arch in the ſaid Rank : To the Sum of the Sines of two Arches next it on each Side; having the ſame common Diſtance.* }

Immediately after theſe Proportions, he lays down the following Equations:

Three Arches equi-different, being propoſed; if (ſaith he) you put Z = the Sine of the greater Extreme, $y =$ the Sine of the leſſer Extreme; $M =$ the Sine of the Mean; $m =$ the Co-Sine thereof; $\mathcal{D} =$ the Sine of the common Difference; $d =$ the Co-ſine thereof; and $R =$ the Radius.

1. Then $Z + y = \frac{2Md}{R}$. 2. Then $Z - y = \frac{2m\mathcal{D}}{R}$.

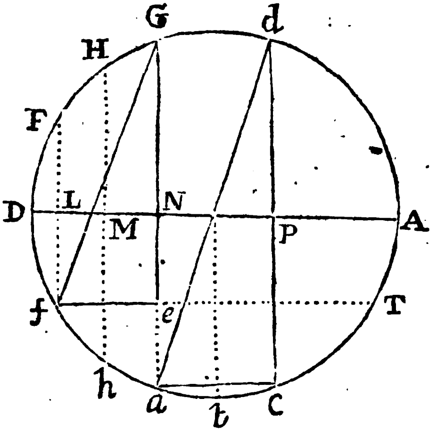
3. Then $Zy = MM - \mathcal{D}\mathcal{D}$. 4. Then $\frac{Z}{y} = \frac{Md + m\mathcal{D}}{Md - Md}$

From the foregoing, it is evident, (ſaith he) that if two Thirds, *viz.* either the former or latter 60 Degrees, or the former 30 Degr. and the latter 30 Degr. of the Quadrant be compleated with Sines; the remaining part of the Quadrant may be compleated by Addition, or Subduction only.

Thus far is from the Ingenious Mr. *Dary*, concerning theſe excellent Proportions; the Truth whereof I ſhall thus demonſtrate.

In the annexed Circle $DA = da$ are Diameters, $fh = ha = ab = bc$, are equal Arches.

Draw fT parallel to DA; then will $NE = Lf$. And the $\triangle dac$, like the $\triangle Gfe$, being both right-angled at c and e. and $\angle = d$ $\angle G$ becauſe ſubtended by the equal Arches $ac = fa$.



Therefore $da : dc :: Gf : Ge$.

Conſequently $\frac{1}{2}da : dc :: \frac{1}{2}Gf : Ge$. But $Hh = Gf$, whence $HM = \frac{1}{2}Gf$, and $\frac{1}{2}da$ = the Radius, $dp = \frac{1}{2}dc$. Therefore it will be, Radius : $2dp :: HM = \frac{1}{2}Gf : GN + Ne = GN + Lf$. That is, As the Radius : Is to twice the Sine dp :: So is the Sine HM : To the Sum of the two Sines GN and $FL = fL$. Q. E. D.

I ſhall now explain theſe Proportions, and ſhew how they may be applied in Practice: Having the Sine of one Minute, and its Co-Sine as before; let the Radius be made the Mean or Middle Term between thoſe two Extremes; then the Proportions will run

Thus, *As the Radius : Is to the double Co-Sine of one Minute :: So is the Sine of one Minute : To the Sine of two Minutes, and of 00′ : And ſo is the Sine of 2′ :: To the Sum of the Sines of 3′ and 1′ :: And ſo is the Sine of 3′ : To the Sum of the Sines of 4′ and 2′.*

And ſo on in a ſucceſſive Order, from Minute to Minute.

And then, if from the Sum of the Sines of 3′ and 1′ be taken the Sine of 1′, the Remainder will be the Sine of 3′: And the like, if from the Sum of the Sines of 4′ and 2′, be taken the Sine of 2′, there will remain the Sine of 4′, &c.

Proceeding on by this Method, all the Natural Sines in the Quadrant may be eaſily calculated by Addition, and Subſtraction only. For the Radius, or firſt Term in the Proportion, being 1,0000000 or Unity, Diviſion is wholly eſcaped: And becauſe the ſecond Term in the Proportion varies not, if a Tariffa, or ſmall Table be made thereof, to all the nine Digits, then Multiplication is alſo eſcaped. For by the help of that Tariffa, the whole Work may be perform'd by Addition and Subſtraction, until all the Sines are gradually made.

Thus you have an eaſy way of making the Canon of Sines; which being once done, the Tangents may be found by the following

Proportion, { *As the Co-Sine of any Arch* : *Is to the Sine of that Arch* :: *So is the Radius* : *To the Tangent of the ſame Arch.*

That is, by *the firſt Scheme of this Problem*, $RN : SN :: RA : TA$. And $RN : RS :: RA : RT =$ the Secant of that Arch.

I ſhall now proceed to a ſecond Method of finding the Natural Sine of any propoſed Arch or Angle at pleaſure, by the Solution of two Equations only, without the aſſiſtance of any precedent Sines (as above) ſave only that of 1′, *viz.* 0,000290888, found as at firſt from the Periphery of the Circle whoſe Radius is Unity or 1. This Method of mine, (*for ſo I may juſtly call it*) admits of two Caſes.

Caſe 1. If the propoſed Arch (*or Angle*) be leſs than 45 Degrees, reduce the Degrees into Minutes, and then multiply thoſe Minutes into this conſtant Multiplicator 0,000290888, and call that Product p, and for the Sine ſought put a. Then it will always be this general Equation,

$$-aaaa + 12paaa - 195aa - 36ppaa + 240pa = 45pp.$$

Example 1.

Suppoſe it were required to find the Sine of 17 deg. 10′.

Firſt, 17° 10′ = 1030′. Then ,000290888 × 1030 = 299614, &c. $= p$. Thence the Equation, with all its Co-efficients, will become, $-aaaa + 3{,}595375aaa - 198{,}23168aa + 71{,}9076136a = 4{,}03960196$ the Reſolvend.

N. B. In this Caſe, the firſt r may always be taken the next neareſt to the firſt Figure in the Product $= p$. That is, I here take $r = 0{,}3$ being more than juſt, therefore it will be $r - e = a$.

$$\text{Conſequently} \begin{cases} rr - 2re + ee = aa \\ rrr - 3rre + 3ree = aaa \\ rrrr - 4rrre + 6rree = aaaa \end{cases}$$

This $r = 0{,}3$ being involved as its Powers direct, and thoſe Powers multiplied into their reſpective Coefficients of the laſt Equation above, will produce

$$\begin{array}{lllll} +21{,}57228 & -71{,}9076e & & & = a \\ -17{,}84085 & +118{,}9390e & -198{,}2316ee & & = aa \\ +\ 0{,}09707 & -\ 0{,}9707e & +\ 3{,}2358ee & & = aaa \\ -\ 0{,}00810 & +\ 0{,}1080e & -\ 0{,}54000ee & & = aaaa \\ \hline \textit{viz.}\ +21{,}6693 & +46{,}1687e & -195{,}5358ee & & = 4{,}03960196 \end{array}$$

Hence it will be $0{,}2361e - ee = 0{,}00112103 = \mathcal{D}$.

$$\textit{Theorem} \left\{ \frac{\mathcal{D}}{0{,}2361 - e} = e \right.$$

$$\begin{array}{lrlr} \text{Operation} & 0{,}2361 &)\ \dot{0}{,}\dot{0}0\dot{1}1\dot{2}103 & (0{,}00483 \\ & -e = \quad 48 & \quad 928 & \\ \cline{2-3} \text{Firſt Diviſor} & 0{,}232 & \quad 19303 & \\ \cline{2-2} \text{Second Diviſor} & 0{,}2313 & \quad 18504 & \\ \cline{3-3} & & \quad\ \ 7990 & \end{array}$$

$$\begin{array}{l} \text{Firſt } r = 0{,}3 \\ \quad -e = 0{,}00483 \\ \hline \end{array}$$

$r - e = 0{,}29517 = a$. Or rather New $r = {,}2951$ for a ſecond Operation.

If the Powers of this new $r + e$ be raiſed, and thoſe wherein ee is concern'd be wholly rejected, the remaining Powers being multiplied into their reſpective Co-efficients (as before) will produce the following Numbers, from whence the Value of e may be found by plain Diviſion only.

$$\left.\begin{array}{rr} +21,21993653 & +71,90761e\\ -17,26280771 & -116,99634e\\ +0,09239572 & +0,93989e\\ -0,00758624 & -0,10279e \end{array}\right\}4,03960196$$

$$\overline{+4,04193836-44,2516e=4,03960196}$$

That is $+44,2516e=0,00233640=\mathfrak{D}$

Conſequently $\left\{\frac{\mathfrak{D}}{44,2516}=e\right.$

Operation $44,2516)\ 0,002336400\ (0,0000527=e$

$$\begin{array}{r} 2212580\\ \hline 1238200\\ 885032\\ \hline 3531680\\ 3097612\\ \hline \text{\&c.} \end{array}$$

Laſt $r=0,2951$

$+e=0,0000527$

$r+e=\ ,2951527=a$ the Natural Sine of 17° 10′ as was required.

Example 2.

Let it be required to find the Natural Sine of 25° 32′ $=1532'$, Then $,000290888\times1532=0,445640416=p$. For the Sine ſought put a; as before in the laſt Example.

Let $r+e=a$. And let $r=0,4$ the firſt Figure in the Product $=p$. The general Equation in this Caſe is $-a^4+12pa^3-195aa+36ppaa+240pa=45pp$; as before.

And becauſe it's now, $r+e=a$

Therefore, it will be, $rr+2re+ee=aa$

And $\left\{\begin{array}{l} rrr+3rre+3ree=aaa\\ rrrr+4rrre+6rree=aaaa \end{array}\right.$

Theſe Powers of $r=0,4$ being truly raiſed, and multiplied into the Co-efficients, will produce

$$\left.\begin{array}{rrr} +42,7815 & +106,9537e & \\ -32,3439 & -161,7195e & -202,1494ee\\ +0,3422 & +2,5669e & +6,4172ee\\ -0,0256 & -0,2560e & -0,9600ee \end{array}\right\}=8,9368$$

viz. $+10,7542-52,4549e-196,6922ee=8,9368$

Hence $+52{,}4549e+196{,}6922ee=1{,}8174$
Which being divided by $196{,}6922$
will become $0{,}266e+ee=0{,}009240=\mathcal{D}$

$$\text{And}\left\{\frac{\mathcal{D}}{0{,}266+e}=e\right.$$

Operation $0{,}266)\ 0{,}009240\ (,031=e$
$+e=31$ $\qquad 87$
Firſt Diviſor 29 $\qquad 540$
Second Diviſor 297 $\qquad 297$

Firſt $r=0{,}4$
$+e=\ ,031$
$r+e=0{,}431=r$ for a ſecond Operation.

Which being involved, *&c.* as in the laſt Example, will produce theſe following Numbers:

$$64{,}6392e=0{,}002416118$$

Thence it will be $64{,}6392)\ ,002416118\ (,0000373=e$
$\qquad 1939176$
Laſt $r=0{,}431$ $\qquad 4769420$
$+e=0{,}0000373$ $\qquad 4524744$
$r+e=\ ,4310373=a$ &c. $\qquad 244676$

Caſe 2. When the propoſed Arch or Angle is greater than 45 Degrees, you muſt take its Complement to 90°, *viz.* Subſtract it from 90 Degrees, and then reduce the Remainder into Minutes, as before: And always multiply the Square of thoſe Minutes into this conſtant Multiplicator 0,0000000084616, and call their Product p, putting $a=$ the Sine ſought.

Then calculate by the following Theorem:

$$a^4+28a^3+195aa+36paa+108pa-28a=196-81p$$

Example.

Let it be required to find the Sine of 68° 20′; or, (which is the ſame thing) the Co-Sine 21° 40′ = 1300′. Its Square, $1690000\times0{,}0000000084616=0{,}14300106=p$. Hence the Equation in Numbers will be $a^4+28aaa+200{,}14804aa-12{,}5559a=184{,}41691576$, the Reſolvend.

N. B. In this ſecond Caſe, I take the firſt $r = 1$, and then it will be $r - e = a$, &c. as in the firſt Example: which involved and multiplied as in that Example, will produce theſe Numbers;

viz. $216{,}5921 - 475{,}74e + 290{,}148ee = 184{,}4169$ &c.

Which being duly ordered, will become this Equation, $+475{,}74e - 290{,}148ee = 32{,}17518$.

Divide all by $290{,}148$, and it will be reduced to $+1{,}639e - ee = 0{,}110892 = D$.

Theorem. $\left\{ \frac{D}{1{,}639 = e} = e \right.$

· · ·

Operation $1{,}639)\ 0{,}110892\ (,071 = e$

$-e = 71$ 1092

First Diviſor 156 1692

Second Diviſor 1568 1568

Firſt $r = 1$,

$-e = 0{,}071$

$r - e = {,}929 =$ New r for a ſecond Operation.

Then putting $r + e = a$, which being duly involved, and multiplied as the Equation requires, will produce theſe following Numbers:

$$\left.\begin{array}{llll} - & 11{,}6644218 & - 12{,}55589e & \\ + & 172{,}7359646 & + 371{,}87505e & + 200{,}148ee \\ + & 22{,}4494225 & + 72{,}49536e & + 78{,}036ee \\ + & 0{,}7444839 & + 3{,}20706e & + 5{,}178ee \end{array}\right\} = 184{,}41691576$$

$+184{,}2654992 + 435021 58e + 283{,}362ee = 18441691576$

That is, $+435{,}02158e + 283{,}362ee = 0{,}15141656$

Which reduced by Diviſion, you will have theſe Numbers $1{,}53521e + ee = 0{,}00053436 = D$

Then $\frac{D}{1{,}53521 + e} = e$

· · · ·

Thus $1.53521)\ 0{,}00053436\ (0{,}0003480 = e$

$+e = 34$ 46065

Firſt Diviſor 1.5355 737100

Second Diviſ. 1.53555 614220

1228800

1228440

360

Laſt $r = ,929$
$+ e = ,0003480$
$r + e = ,9293480 = a_r$ the Sine of 68° 20′ *prope.*

Having both the Sine and Co-Sine of any Arch given, the Tangent of that Arch may be found by the Proportion already laid down in this Problem.

But if either the Sine, or the Co-Sine be given, the Tangent may be as eaſily found by the following Theorems. Thus

Let $S =$ the Sine; and $Z =$ the Co-Sine; $T =$ the Tangent; and $1 =$ the Radius.

Then S being given, T may be found by this

$$\textit{Theorem}\left\{\sqrt{\frac{ZZ}{1 - ZZ}} = T.\right.$$

And if Z be given, then T may be found by this

$$\textit{Theorem}\left\{\sqrt{\frac{ZZ}{1 - ZZ}} = T.\right.$$

Example.

Suppoſe $S = ,2951527$ the Sine of 17° 10′ found by our firſt Example before.

Firſt $,2951527 \times ,2951527 = ,087115116 = SS.$
Next $1 - ,087115116 = ,91288484 = 1 - SS$
Then $,91288484)\ ,087115116\ (,09542836$
And $\sqrt{,09542836} = ,3089147 = T$, the Tangent of 17° 10′; as was required,

There are other Varieties of Proportions betwixt the Sines, Tangents, Secants, and their Complements, which I ſhall not touch upon in this place; theſe being ſufficient to ſhew how the whole Canon of Natural Sines and Tangents, or any part thereof, may be made, from the Effects of the tenth Problem.

Having once made the Canon of Natural Sines, *&c.* it will not be difficult to compoſe the Artificial Sines, *&c.* they being only the Logarithms of the Natural, due reſpect being had to the Characteriſtick of the Radius, which is always made 10. As tho' the Radius of the Natural were 10000000000, notwithſtanding it may be more or leſs at pleaſure; the Characteriſtick of the Radius being thus aſſign'd, that of the other Sines will be 9;

till it come to the Sine of 5° 44′, and there it changes to 8, &c. according as the Natural Sines decreaſe in Number of Figures.

And as a further Help therein, I ſhall now ſhew the making of Logarithms.

PROB. XIV.

The Conſtruction, or Making of Logarithms.

THESE moſt excellent and uſeful Numbers were firſt invented by the famous and never to be forgotten Lord *Neiper*, Baron of *Merchiſton* in *Scotland*, (*An.* 1614.) who ingeniouſly contrived to perform Multiplication and Diviſion of Natural Numbers, by only adding or ſubſtracting certain artificial Numbers, which he called *Logarithms.*

This Invention of his (no doubt) proceeded from a mature Conſideration of the Coherence that is betwixt Numbers in Geometrical Proportion and thoſe in Arithmetical Progreſſion.

As in theſe following:

$$\textit{Viz.}\left\{\begin{array}{l} 1 \,.\, 2 \,.\, 4 \,.\, 8 \,.\, 16 \,.\, 32 \,.\, 64 \,.\, 128,\ \&c.\ \text{Geometrical,} \\ 0 \,.\, 1 \,.\, 2 \,.\, 3 \,.\, 4 \,.\, 5 \,.\, 6 \,.\, 7,\ \&c.\ \text{Arithmetical.} \end{array}\right.$$

It is very perceptible, That as the Numbers in the Geometrical Proportionals are produced by *Multiplication* or *Diviſion*, thoſe in the Arithmetical Progreſſion are produced by *Addition* or *Subſtraction*: As doth appear by this Example;

$$\textit{Viz.}\left\{\begin{array}{l} 4 \times 32 = 128 \\ 2 + 5 = 7 \end{array}\right\}\text{or}\left\{\begin{array}{l} 128 \div 32 = 4\ \text{Geometr.} \\ 7 - 5 = 2\ \text{Arithmet.} \end{array}\right.$$

$$\text{Again,}\left\{\begin{array}{l} 1 \,.\, 10 \,.\, 100 \,.\, 1000 \,.\, 10000 \,.\, 100000,\ \&c.\ \text{Geometr.} \\ 0 \,.\, 1 \,.\, 2 \,.\, 3 \,.\, 4 \,.\, 5\ \ \&c.\ \text{Arithmet.} \end{array}\right.$$

The ſame Coherence is betwixt theſe latter, as was between the two firſt Ranks.

$$\textit{Viz.}\left\{\begin{array}{l} 1000 \times 10 = 10000. \\ 3 + 1 = 4 \end{array}\right\}\text{or}\left\{\begin{array}{l} 100000 \div 1000 = 100\ \text{Geom.} \\ 5 - 3 = 2\ \text{Arith.} \end{array}\right.$$

Either of theſe Examples do ſufficiently ſhew the Reaſon and very Ground of Logarithms.

And from the latter of theſe it was, that the Prime Logarithms or Characteriſticks were firſt aſſigned.

As in this Table:

Natural Num.	*Logarithms.*
1	0,0000000
10	1,0000000
100	2,0000000
1000	3,0000000
10000	4,0000000
100000	5,0000000

Having laid this Foundation, the next Work was to find out the Logarithms of the intermediate Numbers ſituated betwixt 1 and 10, *viz.* of 2, 3, 4, 5, 6, 7, &c. and of thoſe betwixt 10 and 100, *viz.* of 11, 12, 13, 14, 15, &c. and ſo on for the reſt. This was a Work of ſome difficulty, and very laborious.

The firſt Step in order thereunto (as I conceive) was to find out a Rank of continual Means betwixt 10 and 1, ſo as that the laſt (and leaſt thereof) might be a mixed Number leſs than 2, and ſo near 1, as to have ſuch a number of Cyphers before the ſignificant Figures thereof, as was intended the Places of Logarithms in the Table ſhould conſiſt of. Which Means are to be found, by extracting the Square Root of 10 (having firſt annexed a competent Number of Cyphers thereunto;) then extracting the Root of that Root, and ſo by a continued Extraction of Root out of Root, until there be a Root ſo qualify'd as before-mentioned: Which to make a Table to ſeven Places in the Logarithms, will require twenty-five ſeveral Extractions, the laſt of which will produce this Number, 1,00000006862238.

The next Step was to find out a Number betwixt (1) and (0) in Arithmetical Progreſſion, that might truly correſpond with the Mean before found (betwixt 10 and 1) ſuch a Number muſt conſequently be its Logarithm. And this may be found by a continual biſecting (or halving) of 1, ſo often as was the Number of the foregoing Extractions, (to wit, twenty-five) the laſt of which Biſections will produce 0,000000029802322, &c. the true Logarithm of 1,00000006862238.

For as 1,00000006862238 by twenty-five continued Involutions (*viz.* firſt into itſelf, then that Product into itſelf, and ſo on ſucceſſively) will produce 10; ſo will 0,000000002980232 by the like Number of doublings and re-doublings, produce 1.

This Mean (or Number) and its Logarithm being thus found, it will follow by Proportion,

As the Significant Figures of this Mean : *Are to the Significant Figures of its Logarithm* :: *So are the Significant Figures of any Mean, betwixt any given Number and* 1 : (having ſeven Cyphers before ſuch Figures, as this hath) *To the Significant Figures of its Logarithm.* To which muſt be prefixed ſeven Cyphers to compleat it. After which, being doubled, and redoubled according to the Number of Extractions required to produce its correſponding Mean, will at laſt diſcover the true Logarithm of the given Number. For the clearing of this, take an Example.

Suppoſe it were required to find the Logarithm of the Number 2, to ſeven places. Firſt, by a continued Extraction of Root out of Root, beginning at 2, find ſuch a Mean (or Root as before) betwixt 2 and 1, as will have ſeven Cyphers before its Significant Figures; which after twenty-three ſeveral Extractions, will be this Number 1,00000008262958. Then according to the foregoing Proportions, it will be

6862238 : 2980232 :: 8262958 : 3588557

To which prefix ſeven Cyphers, as before directed, then will 1,00000008269958 have for its Logarithm, ,00000003588557; which being doubled and redoubled (as aboveſaid) will produce 0,30102997958658 the true Logarithm of 2; which being contracted to ſeven places, according to the firſt Deſign (and agreeable to the ſeven Places of Cyphers) then it will become 0,3010299. But in all the Tables that I have ſeen, the Logarithm of 2 is 0,3010300: I conceive the reaſon is, becauſe the remaining Figures 7958658 come ſo near Unity of the laſt place in the retained Figures.

And by the ſame Method that this Logarithm of 2 is made, may the Logarithm of any other Number be found. But when once the Logarithms of a few of the prime Numbers, *viz.* of 3. 7. 11. 13. &c. (that is, of ſuch Numbers as cannot be produced by the multiplying of two Integer Factors) are obtained, the reſt may be eaſily compoſed by *Addition* and *Subſtraction* only.

For as $3 \times 2 = 6$ So Log. of $3 +$ Log. of $2 =$ Log. of 6.

And as $10 \div 2 = 5$ So Log. of $10 -$ Log. of $2 =$ Log. of 5

The like of all Numbers that have Aliquot Parts, (that is, ſuch Integer Numbers as may be divided by Integers.) And indeed the Logarithms of ſeveral of the Prime Numbers may alſo be obtained by *Addition* or *Subſtraction*, as might eaſily be ſhewed, and is not difficult to conceive by any one, who but duly conſiders the Nature and Deſign of Logarithms; as will in a great meaſure appear by their Uſe in the following Part of Intereſt, &c. of which I ſhall forbear ſaying any thing in this place, and keep to my firſt Deſign herein, which was to give a brief Account of the Ingenious Author's Method (as I conceive it) of making the ſame: who undoubtedly found it a very difficult Work, by reaſon there is required ſo many ſeveral Extractions of Roots out of Roots, which muſt needs render it both troubleſome and laborious. Then to propoſe a different Method of raiſing the Logarithms of ſuch Prime Numbers before mentioned (which require the Extraction of Roots to obtain their reſpective Means) with one tenth part of the Trouble and Time required by the foregoing Method, And not only ſo, but more exact; for by our preſent Methods of Converging Series, the Root of any Power, how high ſoever it be, is eaſily found at one ſingle Extraction; and thereby the Errors which would ariſe by Extracting a *Surd Root* out of a *Surd Root* (eſpecially when often repeated) are avoided; and conſequently ſuch a Mean as may be required betwixt any Number and Unity, is thereby more exactly found.

Now how this may be performed, I here intend to ſhew, as briefly as I can. In order thereunto, take this as a Model.

Let $a =$ the Root, or Mean required betwixt any Number and Unity:

$$\text{Then}\left\{\begin{array}{lll} a^{2} = \square a. & a^{4} = \square a^{2}. & a^{8} = \square a^{4}. \\ a^{16} = \square a^{8}. & a^{32} = \square a^{16}. & a^{64} = \square a^{32}. \\ a^{128} = \square a^{64}. & a^{256} = \square a^{128}. & a^{512} = \square a^{256}. \end{array}\right.$$

And ſo on ſucceſſively with the Indices in Geometrical Progreſſion, until the Power of a be made equal to ſuch a Term in that Progreſſion, as that the Root, or Value of a may have betwixt Unity and its ſignificant Figures, ſo many Cyphers, as are the intended Number of Places in the Logarithms.

For inſtance, let it be required to find the Mean between 10 and 1, (as in *pag.* 111.) then the Power of a muſt be $a^{33554432} = 10$, this Index 33554432 being the 25th Term in Geometrical Progreſſion, which may be thus determined.

Let 1, the Characteriſtick or Logarithm of 10, be divided by ſuch a Term in Geometrical Progreſſion, as will cauſe ſuch a Number of Cyphers to be before the ſignificant Figures in the Quotient, as are required to be before the Figures of the Root a; ſuppoſe 7 (as before)

Then $1 \div 33554432 = ,00000002980232$, &c.

which is the true Arithmetical Mean (as before found, by a continual biſecting of 1) correſpondent to that ſignified by a. And therefore the Value of a, found by extracting the reſpective Root of $10 = a^{33554432}$ will be the Mean required;

Viz. 1,00000006862238 whoſe Log. is ,00000002980232

Theſe being found, are the Foundation of the reſt, as before.

Then ſuppoſe it be required to find the Logarithm of any of the Prime Numbers; if you pleaſe, that of 2. In order thereunto, let $a =$ the Root, or Mean ſought betwixt 2 and 1 (as before;) then muſt a be continually involved (as by the above Model) until its Index be equal to the greateſt Term in Geometrical Progreſſion, whoſe Number of Places of Figures are to be equal to the Number of required Cyphers before a; to wit 7. According to which, the Power of a will be $a^{8388608} = 2$ (this 8388608 being the 23d Term in Geometrical Progreſſion) conſequently the reſpective Root of $2 = a^{8388608}$ will be the Mean required.

Example.

Let $r + e = a$

Then will $r^{8388608} + 8388608\, r^{8388607} e$

$+ 35184367894528\, r^{8388606} ee = a^{8388606} = 2$

Suppoſe $r = 1$

Then $1 + 8388608e + 35184367894528ee = 2$

That is $8388608e + 35184367894528ee = 1$

Each part being divided by the Co-efficient found prefixed to *ee*, viz. 351843, &c. then it will become

$$,00000023e+ee=,0000000000000284=\mathcal{D}$$

Conſequently $\left\{\dfrac{\mathcal{D}}{,00000023+e}=e\right.$

. . . .

$,0000000000000284=\mathcal{D}$

$,00000023$ $\qquad$ $\underline{248}$ (,00000008 $=e$

$+e=\underline{,00000008}$ $\qquad$ 36

Diviſor ,00000031

Firſt $r=1,$

$+e=\underline{,00000008}$

New $r=1,00000008$

Which being duly involved (in the ſame Order as the Model denotes) and multiplied into the reſpective Coefficients, will then produce theſe Numbers,

viz. $1,9563638967+164111168e+6883341606628 9ee=2$

Then $164111168e+68833416066289ee=,0436361033$

And $,0000002384e+ee=,00000000000000063393=\mathcal{D}$

Conſequently $\left\{\dfrac{\mathcal{D}}{,0000002384+e}=e\right.$

. . . .

$,00000000000000063393=\mathcal{D}$

$,0000002384$ $\qquad$ $\underline{480}$ (,000000000263 $=e$

$+e=\underline{,0000000026}$ $\qquad$ 15393

Diviſor $\underline{,000000240}$ $\qquad$ $\underline{14460}$

Diviſor ,0000002410 $\qquad$ 9330

$\underline{7230}$

Laſt $r=1,00000008$

$+e=\underline{,000000000263}$

New $r=1,000000008263$

I take only $1,00000000286=r$; the which being involved, and ordered as before, will produce theſe following Numbers, *viz.*

$1,999503684867+167730028e+70351267454084ee=2$

Then $167730028e+70351267454084ee=,000496315133$

$,0000002384186e+ee=00000000000000000705481443=\mathcal{D}$

Conſequently $\left\{ \frac{D}{,0000002384186 + e} = e \right.$

,000000000000000000705481443 = *D*

,0000002384186 47686 (,0000000000295 = *e*
+ *e* = ,0000000000295 2286214
Diviſor ,00000023843 2146023
Diviſor ,000000238447 14019143
Diviſor ,0000002384481 * 11922405
*20967380 (879 = *e*
19075848
1891532
1669136
222396
214596

* Here I deſiſt forming a new Diviſor, and make uſe of the Abridgment, as in *pag.* 65, and elſewhere.

Laſt *r* = 1,0000000826
+ *e* = ,0000000000295879
a = 1,0000000826295879

This Value of *a* = 1,0000000826295879 is the Geometrical Mean betwixt 2 and 1, as was required; (agreeable to that before found, by twenty-three ſeveral Extractions.) And by this Method of proceeding, may be found the Mean betwixt 10 and 1, *viz.* 1,00000006862238, or betwixt any other of the (before mentioned) Prime Numbers and Unity, as might eaſily be ſhewed. But for brevity ſake, I ſhall omit giving more Examples thereof, this one being ſufficient (eſpecially to the Ingenious) if well conſidered, and but once underſtood, to ſhew the nature of, and manner how, to proceed upon the like occaſion, of finding any propoſed Mean. The next thing will be to find the Logarithm of the Number from whence ſuch Mean was produced, which may be thus performed.

Firſt, find its correſponding Arithmetical Mean, or Logarithm, by Proportion, (as in *pag.* 112.) Then multiply that correſponding Mean (ſo found) into the Index Number of ſuch Power as the Geometrical Mean was produced from; that Product will be the Logarithm of the given Number, (without a continued doubling and redoubling, as before.) For the clearing of this, let it be required to compleat the Logarithm of 2.

Having firſt found 1,000000006862238, the proper Geometrical Mean betwixt 10 and 1; alſo its correſponding Logarithm ,00000002980232 (as before directed) with them, and the Mean betwixt 2 and 1, laſt found, *viz.* 1,0000000826295879; make uſe of the above-mentioned Proportion, (as in *pag.* 112.) *viz.*

6862238 : 2980232 :: 826295879 : 358855729

To which prefix ſeven Cyphers to compleat it, (as before.) Then it will become ,0000000358855729. This Number being multiplied into the Power of *a* (what that is, ſee *pag.* 115.) will produce the Logarithm of 2.

viz. ,0000000358855729 × 8388608 = 0,30103000391352

But according to the firſt Deſign, it is required to have but ſeven Places, *viz.* 0,301300; which is the true Logarithm of 2 without any defect.

Thus I have preſented you with a New and Expeditious Method of making Logarithms; which if they were required to fourteen or fifteen Places (I can modeſtly ſay) they might then be made with one twentieth part of the Time and Trouble required by the firſt Method.

Before I conclude, it will not be amiſs to inſert an Example or two of ſuch Equations as have all their Terms in them. Suppoſe this Equation were given,

$aaa + baa - da = G$, what's the Value of a?

In Numbers $aaa + 438aa - 7825a = 98508430$

$b = 438 \quad d = 7825 \quad G = 98508430$

Let $r + e = a$;

$$\text{Then}\left\{\begin{array}{llll} rrr & + 3rre & + 3ree & = aaa \\ brr & + 2bre & + bee & = baa \\ -dr & - de & & = -da \end{array}\right.$$

Suppoſe $r = 4$, then $rr = 16$, $rrr = 64$, $brr = 64$, and $dr = 0$; (for there is no Figure under the firſt Point at d.) Then doth $rrr + brr = 128 > 98$ *Ergo* $r < 4$

Let $r = 300$, then it will be

$$\left.\begin{array}{lll} +27000000 & +270000e & +900ee \\ +39420000 & +262800e & +438ee \\ -2347500 & -7825e & \end{array}\right\} = G$$

That is, $64072500 + 524975e + 1338ee = G$.
Then $524975e + 1338ee = 34435930$
And $392e + ee = 25736 = \mathcal{D}$.

Conſequently $\left\{ \frac{\mathcal{D}}{392 + e} = e \right.$

392
$+e = \underline{5}$ $\qquad \dot{2}5\dot{7}3\dot{6} = \mathcal{D}$
Diviſor 44 $\qquad \underline{220}$ $\qquad (5 = e$
$+e = \underline{8}$ $\qquad 3736$ $\qquad (8 = e$
Diviſor 450 $\qquad 3600$

Laſt $r = 300$
$+e = \underline{58}$
New $r = 358$ I take but $350 = r$

This New r being involved and multiplied into the Co-efficients, as the Species above dire&ts, it will be

$+42875000 + 3675000e + 1050ee$
$+53655000 + 3066000e + 438ee$
$-\ 2738750 -\ 7825e$

That is, $93791250 + 666275e + 1488ee = G$
Then $666275e + 1488ee = 4717180$
And $447{,}76e + ee = 3170{,}1478 = \mathcal{D}$.

Conſequently $\left\{ \frac{\mathcal{D}}{447{,}75 + e} = e \right.$

$447{,}76$ $\qquad 3\dot{1}7\dot{0},1\dot{4}7\dot{8} = \mathcal{D}$
$+e = \underline{6{,}99}$ $\qquad \underline{2718}$ $\qquad (6{,}994 = e$
Diviſor $\underline{453,}$ $\qquad 452{,}14$
Diviſor $\underline{454{,}6}$ $\qquad \underline{409{,}14}$
Diviſor $454{,}75$ $\qquad 430078$
$\qquad \underline{409275}$
$\qquad 208030$

Laſt $r = 350,$
$+e = \underline{6{,}994}$
$a = 356{,}994$

In this Proceſs, I humbly conceive, there is no difficulty appears; that which ſeemeth like one, is the placing of the firſt Value of $e = 5$. But if it be conſidered, that there muſt be three Places of Integer Figures in the Root (as appears by the Points) and that this 5 is to be the ſecond Figure thereof, it will be eaſy to determine that its place muſt be under the ſecond Figure of the Diviſor, *viz.* 9.

This Conſideration will be very helpful in all Caſes where the Root conſiſteth of moſt Integers: As for Decimal Figures, they place themſelves.

PROB. XV.

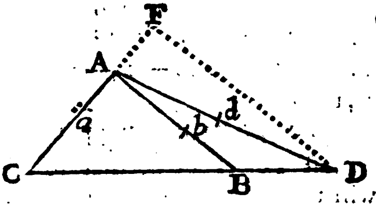
IN the Oblique Triangle *CAD*, there is given the Side *AD*, and the Sum of the other two Sides, *viz.* $CA+CD$; alſo within the Triangle, there is given the Line *AB*, ſo drawn, as to make the included Triangle *CAB* right-angled at *A*.

Thence to find out the Side *CA*.

Let

$CA+CD=z$ }
$AD=d$ } given.
$AB=b$ }

$z=51$
$d=32$
$b=21$



Draw the Line *DF* parallel to *AB*, and continue the Side *CA* to *F*; then the Triangles *CAB* and *CFD* will be alike.

Let $CA=a$ $AF=e$ and $FD=y$

Then	1	$z-a=CD$, by the Prob.
$1 \odot 2$	2	$zz-2za+aa=\square CD$.
but	3	$zz-2za+aa=aa+2ae+ee+yy$
$3-aa$	4	$zz-2za=2ae+ee+yy$
but	5	$dd=ee+yy$
$4-5$	6	$zz-2za-dd=2ae$
Let	7	$2f=zz-dd$
6, 7,	8	$2f-2za=2ae$
$8 \div 2$	9	$f-za=ae$
	10	$aa+bb=\square CB$, by the Scheme.
10 ω	11	$\overline{aa+bb}=CB$.

But

But	12	$CB : CA :: CD : CF$.
That is	13	$\sqrt{aa+bb} : a :: z-a : a+e$
13 ∵	14	$za - aa \div \sqrt{aa+bb} = a+e$
$9 \div a$	15	$f - za \div a = e$
$15 + a$	16	$f - za + aa \div a = a+e$
14, 16,	17	$\frac{za - aa}{\sqrt{aa+bb}} = \frac{f - za + aa}{a}$
$17 \times a$	18	$\frac{zaa - aaa}{\sqrt{aa+bb}} = f - za + aa$
18 ×	19	$za^2 - a^3 = \overline{f - za + a^2} \times \sqrt{aa+bb}$
Then	20	$\frac{za^2 - a^3}{f - za + aa} = \sqrt{aa+bb}$
And	21	$\square\, \frac{zaa - aaa}{f - za + aa} = aa + bb$

This 21ſt Step, or laſt Equation, being firſt involved, then brought out of the Fraction, and reduced into Numbers, will produce this Equation:

$$-2018aaaa + 125609aaa - 2464230{,}25aa + 35468307a = 274183922{,}25$$

Each Member thereof being divided by 2018, the Factor of the higheſt Power of *a*, that ſo the ſame may be cleared, will bring down the Equation to this:

$$-aaaa + 62{,}1aaa - 1221{,}12aa + 17575{,}969a = 135869{,}1388$$

Let the Equation be made

$$-aaaa + baaa - daa + fa = G.$$

Here $62{,}1 = b$, $1221{,}12 = d$
and $17575{,}969 = f$ $135869{,}1388 = G$.

Put $r + e = a$, then

$$\begin{array}{llll} -rrrr & -4rrre & -6rree & = -a^4 \\ +brrr & +3brre & +3bree & = +ba^3 \\ -drr & -2dre & -dee & = -daa \\ +fr & +fe & \ldots\ldots & = +fa \end{array}$$

Let $r = 10$, then it will be

$$\left.\begin{array}{l} -\ 10000 - 4000e - 600ee \\ +\ 62100 + 18630e + 1863ee \\ -\ 122112 - 24422e - 1221ee \\ +\ 175759 + 17575e \ldots\ldots\ldots \end{array}\right\} = G.$$

That is, $105547 + 7783e + 42ee = 135869$

Then $7783e + 42ee = 30322$

And $185{,}6e + ee = 721{,}95 = \mathcal{D}.$

$$\textit{Consequently}\left\{\frac{\mathcal{D}}{185{,}6 + e}\right. = e$$

$$\begin{array}{lll} & 185{,}6 & 721{,}95 = \mathcal{D} \\ +\ e = & \underline{3{,}8} & \underline{564} \qquad\qquad (3{,}8 = e \\ \text{Divisor} & \underline{188} & 15795 \\ \text{Divisor} & 189{,}4 & 15152 \end{array}$$

First $r = 10$,

$+\ e = \underline{3{,}8}$

New $r = 13{,}8$ I take but two places, to wit, $r = 13$; which being involved, and duly multiplied into the reſpective Co-efficients, will produce theſe following Numbers:

$$\textit{Viz.}\left\{\begin{array}{l} -\ 28561{,}00 - 8788{,}00e - 1014{,}00ee \\ +\ 136453{,}70 + 31584{,}70e + 2421{,}90ee \\ -\ 206369{,}28 - 31749{,}12e - 1221{,}12ee \\ +\ 228487{,}60 + 17375{,}97e \end{array}\right.$$

That is, $129991{,}02 + 8623{,}55e + 186{,}78ee = G$

Then $8623{,}55e + 186{,}78ee = 5878{,}1188$

And $46{,}169e + ee = 31{,}470814 = \mathcal{D}.$

$$\textit{Consequently}\left\{\frac{\mathcal{D}}{46{,}169 + e}\right. = e$$

$$\begin{array}{lll} & 46{,}196 & 31{,}470814 = \mathcal{D} \\ +\ e = & \underline{{,}673} & \underline{28{,}02} \qquad\qquad ({,}6736 = e \\ \text{Divisor} & \underline{46{,}7} & 3{,}4508 \\ \text{Divisor} & \underline{46{,}83} & \underline{3{,}2781} \\ \text{Divisor} & 46{,}842 & 172714 \\ & & \underline{140526} \\ & & 321880 \\ & & 281052 \end{array}$$

Laſt $r = 13,$

$+e = \quad ,6736$

$a = 13,6736$

This Problem I invented, and reſolved about ſixteen Years ago (living then at *Cheſter*) from whence it was ſent to a Mathematical Profeſſor in *London*, from whom I was promiſed to receive a Solution thereof; but never did, either from him, or others, until I met with it in Mr. *Raphſon*'s before mentioned *Analyſis*, pag. 24. where the ſame is brought to an Equation by a different Proceſs to what I then did, and have here done.

This I thought convenient to inſert, not for that I ſuppoſe this Problem ſo very difficult (tho knotty enough) but becauſe I have here a fit opportunity to ſignify to the Reader, that I am not a New Pretender to this Excellent Art, having been converſant in it, and other Parts of the Mathematicks, near twenty Years; yet never thought it proper to appear in publick, until I had ſomething to preſent the World with, worth (as I humbly conceive) their acceptance: Whether this Tract be ſuch, or not, I leave to the Conſideration of the Ingenious.

PART II.

OF

Intereſt and *Annuities*, &c.

CHAP. I.

The Definition and Uſe of Logarithms *in general.*

IN the Firſt Part of this Treatiſe (*viz.* in the Algebra Part, at *Prob.* 9.) I have ſhew'd the making of *Logarithms.* I ſhall now ſhew their Uſe in performing Multiplication, or Diviſion of Natural Numbers, by Addition, or Subſtraction only.

That is, if any two given Numbers are either to be multiplied or divided, the reſpective Logarithms of thoſe Numbers being accordingly added, or ſubſtracted, their Sum, or Difference, will be the Logarithm of that Natural Number, which is the Product or Quotient of ſuch Multiplication or Diviſion.

And that the Value of any Product, or Quotient, ſo found by the Sum or Difference of two Logarithms, may be truly known, every Logarithm of the Prime Numbers, *viz.* of 1 . 10 . 100 . 1000 . 10000 . *&c.* in whole Numbers :

And of 0,1 . 0,01 . 0,001 . 0,0001 *&c.* in Decimal Parts, is the Index or Characteriſtick to all the intermediate Logarithms which are between them : As in this

TABLE.

Whole Num.	*Logarithms.*	*Decimal Parts.*	*Logarithms.*
1	0.000000	1	0.000000
10	1.000000	0,1	$\bar{9}$.000000
100	2.000000	0,01	$\bar{8}$.000000
1000	3.000000	0,001	$\bar{7}$.000000
10000	4.000000	0,0001	6.000000
&c.	&c.	&c.	&c.

This Table ſhews ſome of the principal *Logarithms*, or *Characteriſticks*, both of whole Numbers and of Decimal Parts, with ſo many Cyphers annexed to each of them (*like Decimal Parts*) as the Number of Places are in the following Table of *Logarithms*. From whence it will be eaſy to perceive, That,

If 0, be made the Logarithm of 1 ; and 1, be made the Logarithm of 10 : Then all the intermediate Logarithms which belong to the natural Numbers, between 1, and 10, (viz. *the Logarithms of* 2. 3. 4. 5. 6 &c.) muſt needs be Decimals leſs than 1. And for the ſame reaſon, That,

If 1. be the Logarithm of 10. and 2. be made the Logarithm of 100. then all the intermediate Logarithms, which belong to the natural Numbers between 10. and 100. will be 1, with Decimal Parts annex'd to it : And conſequently all the Logarithms of the natural Numbers between 100 and 1000, will be 2, with Decimal Parts annex'd to it ; and ſo on for higher Numbers, as in the Table.

Now all theſe principal Logarithms, *viz.* 0. 1. 2. 3. 4 *&c.* of whole Numbers, are called *Affirmative Indices*, or *Characteriſticks*.

And thoſe which belong to Decimal Parts or *Fractions*, *viz.* $\bar{9}.\bar{8}.\bar{7}.\bar{6}$ *&c.* are call'd *Negative Indices* or *Characteriſticks* ; being diſtinguiſh'd from the Affirmative Indices by the Negative Sign — ſet over their heads.

For if 0 be the Logarithm of 1, (*as above*) then it follows, that all the Logarithms of Fractions (viz. *of Numbers leſs than* 1) muſt needs be leſs than 0; that is, they muſt be *Negative* Numbers.

From what hath been here ſaid, it will be eaſy to deduce the General *Rule*, by which the Diſtance of the natural Numbers from the Unit's place is always known.

Viz. *That every Index or Characteriſtick is leſs by an Unit or* 1, *than the Number of Places of Figures in the natural Numbers to which it belongs.*

As for inſtance, in the following Numbers, wherein I ſuppoſe 5381 to be a whole Number; whoſe Logarithm is 3.730863. Now if this natural Number 5381 be otherwiſe taken or varied in its places, then the *Logarithms* will ſtand thus:

Natural Numbers.	*Logarithms.*
5381	3.730863
538,1	2.730863
53,81	1.730863
5,381	0.730863
0,5381	$\bar{9}$.730863
0,05381	$\bar{8}$.730863
0,005381	$\bar{7}$.730863 &c.

In theſe Examples you ſee that the Logarithms are all the ſame, ſave only the *Indices* or *Characteriſticks* are alter'd according to the Diſtance of the firſt Figure of the natural Number from the Unit's place; which being once well underſtood, it will be eaſy to find the Logarithm of any given Number in the *Table* of *Logarithms*, and to prefix its proper *Index* to it.

Or if any Logarithm, with its Index, be given; to find its correſpondent Number, ſo far as the *Table* of *Logarithms* extends; and upon occaſion to one or two places further, with a very ſmall trouble.

Sect. 2. *To find the* Logarithm *of any given Number.*

THE firſt Page of the annex'd *Tables* of *Logarithms* contains all the natural Numbers in their proper Order from 1 to 100. And againſt every one of thoſe Numbers is placed its *Logarithm*, with its *Index* or *Characteriſtick* before it.

Thus, againſt the Number 28 is its Log. 1.447158 And againſt the Number 89 is its Log. 1.949390. And ſo on for the reſt.

In the firſt Column of all, the following Pages (*under Num.*) the natural Numbers proceed in their due Order from 100 to 1000. And in the next Column (*under* 0) againſt every one of thoſe Numbers, is the Decimal Part of its Logarithm, without any Index; to which you muſt prefix its proper Index, according as the natural Number you make uſe of requires.

As for inſtance, againſt the Number 856 (*under* 0) is 932474; to which if 2, the Index of 856, be prefix'd, it will become 2.932474 the compleated Logarithm of 856.

The other five Columns of each Page contains the Logarithms of all the Numbers from 1000 to 10000. Thoſe in the left-hand Pages are diſtinguiſh'd on the top of the Columns with the Figures 0. 1. 2. 3. 4. and thoſe in the right-hand Pages with 5. 6. 7. 8. 9. So that to find the Logarithm of any Number between 1000 and 10000, as ſuppoſe of 5468; you muſt look for the three firſt Figures, *viz.* 546, in the firſt Column under *Num.* and for the laſt Figure, *viz.* 8, at the top. Then in the Column under the laſt Figure 8, and right over againſt the three firſt Figures 546 there is 737829; to which if 3, the Index of 5468, be prefixed, you will then have 3.737829, the compleated Logarithm of 5468, as was required: And ſo for any other Logarithm of any propoſed Number not exceeding 10000.

But if the propoſed Number be above 10000 (*which is the Limits of the annexed Table*) then the Logarithm of that Number muſt be found by help of the common Difference of the Logarithms, which is in the laſt Column of every Page under *Diff.*

Thus,

Find the Logarithm of the firſt four Figures of the given Number, without its Index, (as above) and multiply the common Difference which ſtands againſt that Logarithm (under Diff.*) with the other Figures of the given Number, Caſting off ſo many Figures of that Product, as there are in the Multiplicator.*

Then add the remaining Figures of that Product to the Logarithm of the firſt four Figures, and to their Sum prefix the proper Index, and you will have the compleated Logarithm required.

Example.

Suppoſe it were required to find the Logarithm of 698476. Firſt the Logarithm of 6984 is found in the Table (*as above*) to be 844104, and againſt it, under *Diff.* is 62. This 62 being multiplied with 76 (*the other two Figures of the given Number*) produces 4712. *Cut off* the 12 (*viz. the two laſt Figures*) and then *add* the 47 to the *Logarithm* laſt found, and the *Sum* will be 844151; to which prefixing 5 the proper Index of the given Number 698476, it will be 5.844151, the Logarithm of 698476, as was required.

Sect. 3. *To find the* Number *to any given* Logarithm.

OMit the Index or Characteriſtick of the given Logarithm, and then ſeek it in the Table of Logarithms; and if it can be exactly found there, then the Number in the firſt Column (*under Num.*) with that on the top over the Logarithm, will be the Number required. But if the given Logarithm (*without its Index*) cannot be exactly found in the Table, then the proper Number agreeing to that Logarithm, may be found by the help of the common Difference of the Logarithms:

Thus, { *From the given Logarithm ſubſtract the next leſs, and to the Remainder annex Cyphers; and then divide it by the common Difference found againſt the next leſs Logarithm, under* Diff. *and the Quotient will be a Number that muſt be annexed to the Number already found againſt the next leſs Logarithm, according as the Index of the given Logarithm denotes.*

Example.

Suppoſe 5.660279 were a given Logarithm, and it were required to find the natural Number anſwering to it.

Here the Number ſought muſt conſiſt of ſix Places of Figures in whole Numbers, as appears by its Index 5; which being omitted, I ſeek in the Table of Logarithms for 660279, but not finding it exactly there, I take the next leſs to it, *viz.* 660201, which ſtands under 3, and againſt 457: Therefore I conclude, that the firſt four Figures of the Number ſought, muſt be 4573; and the common Difference found againſt 660201 under *Diff.* is 95.

Then from the given Logarithm 660279
I ſubſtract the next leſs, *viz.* 660201
And there remains 78

To which annexing two Cyphers, (*becauſe there is yet wanting two Places of Figures*) it will become 7800; the which being divided by the common Difference 95, the Quotient will be 82: thus, 95)7800(82, which muſt be annexed to 4573, (*found before*) and the Sum will be 457382, the Number that anſwers to the given Logarithm 5.660279, as was required.

Thus one may, without much trouble, find the Logarithm of any given Number (*very near*) although it exceed the Limits of the Table by 1, 2, or 3 Places of Figures; and alſo the Number agreeing to any given Logarithm, without the help of ſuch a Table of *Proportional Parts*, as is uſually inſerted along with the Table of Logarithms for that purpoſe.

Sect. 4. *To perform* Multiplication *by* Logarithms.

THE Multiplication of any two given Numbers together, may be perform'd by Logarithms:

Thus, { *Add the reſpective Logarithms of the given Numbers together, and their Sum will be the Logarithm of the Product required; due regard being had to the true ordering of their Indices, which admits of three Caſes.*

Caſe 1. If the Indices are both affirmative, their Sum, with what ariſes from the Addition of their Logarithms, will be an affirmative Index.

Examples.

	Numbers.	*Logarithms.*	
Multiplicand	7564	3.878752	Add
Multiplier	75	1.875061	
Product	567300	5.753813	

Again,

Multiplicand	75,04	1.878752	Add
Multiplier	7,5	0.875061	
Product	567,300	2.753813	

Caſe 2. If one of the Indices be Affirmative, and the other be Negative; and if their Sum be above 10: then caſt off 10, and the Remainder will be an Affirmative Index. But if their Sum be leſs than 10, it will be a Negative Index.

Examples.

	Numbers.	*Logarithms.*	
Multiplicand	75,64	1.878752	Add
Multiplier	0,75	$\bar{9}$.875061	
Product	56,7300	1.753813	

Again,

Multiplicand	75,64	1.878752	Add
Multiplier	0,0075	$\bar{7}$.875061	
Product	0,5673	$\bar{9}$.753813	

Caſe 3. If both the Indices are Negative, and their Sum be above 10; then caſt off 10, and the Remainder will be a Negative Index. If their Sum be juſt 10, add 1 to it. If it be under 10, add 10 to it, and that Sum will be a Negative Index.

The two laſt Parts of this *Caſe* ſeldom come into practice. I ſhall therefore only give an Example of the firſt.

Example.

	Numbers.	*Logarithms.*	
Multiplicand	0,0347	$\bar{8}$.540329	Add.
Multiplier	0,0236	$\bar{8}$.372912	
Product	0,00081892	$\bar{6}$.913241	

Sect. 5. *Diviſion perform'd by* Logarithms.

TO divide one Number by another, is only the Converſe of the laſt Work, and is perform'd by Logarithms, Thus, { *Subſtract the Logarithm of the Diviſor from the Logarithm of the Dividend, and the Remainder will be the Logarithm of the Quotient; with this Conſideration, That*

When a leſſer Number divides a greater, the Index of the Quotient Logarithm will be Affirmative: But if a greater Number divides a leſſer, then the Index of the Quotient Logarithm will be Negative, and there muſt be 10 added to the Index of the Logarithm of the Dividend, when Subſtraction cannot be made without it.

Examples.

	Numbers.		*Logarithms.*	
Dividend	567300	——	5.753812	Sub.
Diviſor	7564	——	3.878752	
Quotient	75	——	1.875061	

Again,

Dividend	68,6	——	1.836324	Sub.
Diviſor	78,4	——	2.894316	
Quotient	0,0872	——	9.942008	

Again,

	Numbers.		*Logarithms.*	
Dividend	56,78	——	1.753813	Sub.
Diviſor	0,75	——	9.875061	
Quotient	79,64	——	1.878752	

Sect. 6. *To Extract the Square and Cube Roots, &c. by* Logarithms.

If the Logarithm of any given Number be divided by 2, the Quotient will be the Logarithm of the *Square Root* of that Number. And if the Logarithm of any Number be divided by 3, the Quotient will be the Logarithm of the *Cube Root* of that Number: And ſo on for higher Powers.

Examples.

Suppoſe it were required to extract the *Square Root* of 6968, whoſe Log. is 3.843108

Then 2)3.843108(1.921554 This Quotient is the Logarithm of 83,4745 &c. which is the *Square Root* of 6968, as was required.

Again, Let it be required to extract the *Cube Root* of 6968, whoſe Logarithm is 3.843108 as before.

Then 3)3.843108(1.281036 This Quotient is the Logarithm of 19,1 *&c.* which is the *Cube Root* of 6968; as was required.

And in the ſame manner the *Biquadrat Root*, or that of the Fourth Power, may be found, if the Logarithm of the given Number be divided by 4. And if the Logarithm of any given Number be divided by 5, the Quotient will be the Logarithm of the *Surſolid Root*, or that of the 5th Power, and ſo on for the 6th, 7th, 8th, or any propoſed Root of a ſingle Power, how high ſoever it be, provided the Index of the Logarithm of the given Number be Affirmative. But,

The dividing of Logarithms, whoſe Indices are Negative, (by 2, 3. 4. 5. *&c.*) and to determine the true Quotient-Index hath been thought ſo difficult a Work, that the Learned and Ingenious Mr. *Oughtred* contrived a Table on purpoſe to perform it: Vid. *Key of the Mathematicks*, pag. 168.

Now according to the Negative Indices I have here made uſe of, that Work may be very eaſily perform'd,

Thus, *If you are to divide a Logarithm that hath a Negative Index by 2, add* 10 *to the given Index: If by* 3, *then add* 20 *to the given Index: If by* 4, *then add* 30 *to the given Index* &c.

That is, Still increaſing the Index of the given Logarithm with 10, as the Diviſor doth increaſe by a Unit or 1.

Example.

Suppoſe it were required to extract the *Square Root* of 0,054756 whoſe Logarithm is $\overline{8}$.738432

Then 2)$\overline{18}$.738432($\overline{9}$,369216 This Quotient is the Logarithm of 0,234 which is the *Square Root* of 0,054756 as was required.

Again, Let it be required to extract the *Cube Root* of 0,054756 whoſe Logarithm is $\overline{8}$.738432 as before.

Then 3)$\overline{28}$.738432($\overline{9}$.579477 This is the Logarithm of 0,3797 *&c.* which is the *Cube Root* of 0,054756 as was required.

Once more; Suppoſe it were required to extract the ſecond *Surſolid Root*, or that of the ſeventh Power, out of 0,054756 whoſe Logarithm is $\overline{8}.738432$ as above.

Then $7)\overline{68}.738432(\overline{9}.819776$ This is the Logarithm of 0,66035 *&c.* which is the *Root* required, *&c.*

Thus far may ſuffice concerning the Nature and Uſe of the Table of Logarithms in general; which being a little conſider'd of, it will be eaſy to apply them to the following *Calculations*.

CHAP. II.

The Calculation of Queſtions in Simple Intereſt and Annuities; perform'd both by the Pen, and by Logarithms.

BEFORE we proceed to the following Computations, it may be convenient to premiſe a few uſeful things, that will help to explain and ſhorten the Work. And firſt, of a few common *Characters*, viz.

$+$ } Signifies *More*, or the Affirmative Sign of *Addition.*

$-$ } Signifies *Leſs*, or the Negative Sign of *Subſtraction.*

$\times$ } Signifies *Into* or *With*, and is the Sign of *Multiplication.*

$=$ } Signifies *Equal* to, or the Sign of *Equality.*

Log. } Denotes the compleated *Logarithm* of any Number.

And when the *Ratio* of the *Rate* of *Intereſt* is mention'd, it ſignifies only the Simple Intereſt of 1 *l.* for one Year, at any propoſed Rate of Intereſt *per Cent.* which may be thus found by the *Rule of Three.*

As 100 : is to 6 : : ſo is 1 : to 0,06 the *Ratio* of the Rate of 6 *per Cent. per Ann.*

Or, As 100 : is to 7 : : ſo is 1 : to 0,07 the *Ratio* of the Rate of 7 *per Cent. per Ann.* &c.

The which may alſo be found by Logarithms, Thus, { *From the Logarithm of the given Rate of Intereſt, ſubſtract the Logarithm of* 100 (viz. 2.000000) *and the Remainder will be the Logarithm of the Ratio of that Rate.*

Example.

Suppoſe the given Rate of Intereſt to be that of *6 per Cent. per Ann.* Then

The Logarithm of *6* is 0.778151 } Subſtract.
The Logarithm of 100 is 2.000000 }

The Remainder is the Log. $\overline{8}.778151$ of 0,06
That is, 0,06 is the *Ratio* of *6 per Cent.* &c.

And thus may the *Ratio* of any other propoſed Rate of Intereſt *per Cent.* be eaſily obtain'd.

But becauſe 'tis the Logarithms of thoſe *Ratio's*, that are of uſe in the following Calculations relating to Simple Intereſt; I have here annexed a ſmall Table of ſeveral Rates of Intereſt, with their Ratio's, and the Logarithms of thoſe Ratio's.

Rates of Intereſt per Cent.	*Ratio's of thoſe Rates.*	*Logarithms of thoſe Ratio's.*	*Rates of Intereſt per Cent.*	*Ratio's if thoſe Rates.*	*Logarithms of thoſe Ratio's.*
3	0,03	$\overline{8}.477121$	7	0,07	$\overline{8}.845098$
4	0,04	$\overline{8}.602060$	7 ½	0,075	$\overline{8}.875068$
4 ½	0,045	$\overline{8}.653212$	8	0,08	$\overline{8}.903090$
5	0,05	$\overline{8}.698970$	8 ½	0,085	$\overline{8}.929419$
5 ½	0,055	$\overline{8}.740363$	9	0,09	$\overline{8}.954242$
6	0,06	$\overline{8}.778151$	9 ½	0,095	$\overline{8}.977724$
6 ½	0,065	$\overline{8}.812913$	10	0,1	9.000000

Theſe things being premiſed, we may proceed to the following Work; and firſt, of Money forborn at any Rate of Simple Intereſt, *&c.*

Sect. 1. *Of Simple Intereſt.*

ALL Computations in Simple Intereſt are grounded upon a Series of Terms in Arithmetical Progreſſion.

Let { P ſignify any Principal or Sum put to Intereſt.
T the Time of its continuing at Intereſt.
R the Ratio of the Rate of Intereſt *per Cent.*
A the Amount of the Principal and its Intereſt.

If R = the Intereſt of 1 *l.* for 1 Year, as above;
Then 2 R = the Intereſt of 1 *l.* for 2 Years;
And 3 R = the Intereſt of 1 *l.* for 3 Years, *&c.*

But $R + 2R + 3R$, &c. are Terms in Arithmetical Progreſſion, whoſe firſt Term is R; and the Number of all the Terms will be T. Conſequently the laſt Term will always be TR = the Intereſt of 1 *l.* for any propoſed Time ſignified by T. And then it will be

Thus, { *As* 1 *l. is to its Intereſt* :: *So is any Principal or Sum : to its Intereſt.*

That is $1\,l : TR :: P : TRP$ = the Intereſt of P. Then the Principal being added to its Intereſt, it will be $TRP + P = A$ the Amount required, and gives this general

Theorem, $TRP + P = A$.

This Theorem admits of four Caſes, or Variety of Queſtions.

Caſe 1. If P, T, and R, are given, thence to find A. That is, If any Principal, with the Time of its being at Intereſt, and the Rate of Intereſt *per Cent. per Ann.* are given; to find the Intereſt, and the Amount.

This Queſtion I take to be of the moſt general Uſe of any that occurs in the whole Buſineſs of Simple Intereſt; and may be perform'd thus:

First by Common Arithmetick.

Rule. { *Multiply the Principal, the Time, and the Ratio of the Rate, all three together; and their Product will be the Intereſt: To which add the Principal, and the Sum will be the Amount required.*

Example.

What Sum will 567 l. 10 s. *amount to in nine Years, at the Rate of* 5 per Cent. per Annum?

Here is given $P = 567{,}5$ $T = 9$ and $R = 0{,}06$ to find A; which by the Rule is done thus:

First $P = 567,5$ } Multiply. Again, 5107,5 } Multiply.
And $T = \quad 9$ } $R = 0,06$ }

Product 5107,5 306,45 = 306 *l.* 9 *s.*

That is, 306 *l.* 9 *s.* is the Intereſt of 567 *l.* 10 *s.* for nine Years: Then 306 *l.* 9 *s.* + 567 *l.* 10 *s.* = 873 *l.* 19 *s.* = *A*, the Amount required.

The ſame perform'd by Logarithms.

Thus, { *To the Log. of the Principal, add the Log. of the Time, and the Log. of the Ratio; their Sum will be the Log. of the Intereſt: to which add the Principal, &c. as above.*

That is, in the ſame Example;

Thus, $P = 567,5$ its Log. is 2.753966 }
And, $T = 9$, its Log. is 0.954242 } Add
$R = 0,06$ its Log. is 8.778151 }

The Sum is the Logarithm 2.486359 of 306,45

That is, 306 *l.* 9 *s.* is the Intereſt as before, to which the Princip. 567 *l.* 10 *s.* being added,

the Sum is 873 *l.* 19 *s.* = *A*, the Amount requir'd, as above.

Caſe 2. When *A*, *T*, and *R* are given; to find *P*; that is, To find what Principal or Sum, being put to Intereſt any aſſigned Time, will amount to a propoſed Sum in that Time, at any given Rate of Intereſt *per Cent. per Annum*, &c.

First by the Pen only.

Rule. { *Multiply the Time with the Ratio of the Rate, and to their Product add* 1: *Then divide the propoſed Amount by that Sum, and the Quotient will ſhew the Principal required.*

$\frac{a}{tr+1} = p$

Example.

What Principal or Sum of Money, being put to Intereſt for nine Years, will amount to (or raiſe a Stock of) 873 l. 19 s. *at* 6 per Cent. ? &c.

Or thus; *Suppoſe a Debt of* 873 l. 19 s. *were not to be paid until nine Years hence; what would it be worth in ready Money, the Creditor allowing the Rate of* 6 per Cent. *Diſcompt to the Debtor?*

In this Queſtion there is given, $A = 873,95$ $T = 9$ and $R = 0,06$ to find P,

Thus $T= 9$ } Mult. Then $0,54+1=1,54$
$R=0,06$ } And $1,54)873,95(567,5=P$

Product 0,54

That is, $567,5=567$ *l.* 10 *s.* is the Principal (or Ready Money) requir'd.

The ſame may be perform'd by Logarithms.

Thus, *Add the Log. of the Time to the Log. of the Ratio of the Rate, and to the Number found by their Sum, add* 1. *Then if the Log. of that Sum be ſubſtracted from the Log. of the propoſed Amount, there will remain the Log. of the Principal required.*

That is, in the ſame Example,

Thus $T=9$ its Log. is 0.954242 } Add.
And $R=0,06$ its Log. is 8.778151 }

The Sum is this Logar. 9.732393 of 0,45 To which adding 1, it will become 1,45.

Then $A=873,95$ its Log. is 2.941487 } Subſtract.
And 1,54 its Log. is 0.187521 }

Remains the Logarithm 2.753966 of $567,5=P$

That is, 567 *l.* 10 *s.* is the Principal required by the Queſtion, as above.

Caſe 3. Suppoſe P, T, and A, were given; to find R: *viz.* Having any Principal, with the Time of its being at Intereſt, and the Sum it's propoſed to raiſe or amount to in that Time given; thence to find the Rate of Intereſt *per Cent. per Annum.*

Firſt by the Pen only.

Rule. *If the Difference between the propoſed Amount and the Principal, be divided by the Product of the Principal multiplied into the Time, the Quotient will ſhew the Ratio of the Rate of Intereſt required.*

Example.

At what Rate of Intereſt per Cent. *will* 567 l. 10 s. *raiſe a Stock, or Amount to* 873 l. 19 s. *in nine Years time?*

Here is given $P=567,5$ $T=9$, and $A=873,95$ to find R; which by the Rule is

Thus, $873,95 - 567,5 = 306,45$ the *Dividend*.

And $567,5 \times 9 = 5107,5$ the *Diviſor*.

Then $5107,5)\ 306,45\ (0,06 = R$, the Ratio of the Rate of Intereſt:

And as 1 : is to 0,06 : : ſo is 100 : to 6 the Rate of Intereſt *per Cent*. &c. as was required.

Or by Logarithms,

Thus, *From the propoſed Amount ſubſtract the Principal (as above;) then from the Log. of the Remainder, ſubſtract the Sum of the Logarithms of the Principal and the Time, and there will remain the Log. of the Ratio of the Rate*, &c.

That is, in the ſame Example;

$873,95 - 567,5 = 306,45$ its Log. 2.486359

And $P = 567,5$ its Logarith. is 2.753966 } Add
$T = 9$ its Logarith. is 0.954242 }

From the firſt Log. ſubſtr. this Log. 3.708208

And there remains the Logarith. 8.778151 of 0,06

Which ſhews the Rate of Intereſt to be 6 *per Cent. per Annum*, as before by the Pen.

Caſe 4. Having P, R, and A given, to find T: that is, to find the Time in which any given Principal will amount to, or raiſe any propoſed Sum, the Rate of Intereſt *per Cent*. being alſo given.

Firſt by the Pen only.

Rule. *If the Difference between the propoſed Amount and the Principal, be divided by the Product of the Principal multiplied into the Ratio of the Rate; the Quotient will ſhew the Time required.*

Example.

In what Time will 567 l. 10 s. *amount to (or raiſe a Stock of)* 873 l. 19 s. *at* 6 per Cent. per Annum?

In this Queſt. there is given $P = 567,5$ $R = 0,06$ and $A = 873,95$ to find T; which is done thus:

Firſt $873,95 - 567,5 = 306,45$ the Dividend.

And $567,5 \times 0,06 = 34,05$ for the Diviſor.

Then $34,05)\ 306,45\ (9 = T$ the Time; *viz*. nine Years will be the Time required.

The ſame perform'd by Logarithms,

Thus, { *From the propoſed Amount ſubſtract the Principal (as before;) then from the Logarithm of the Remainder, ſubſtract the Sum of the Logarithms of the Principal and Ratio of the Rate; and there will remain the Logarithm of the Time.*

That is, in the laſt Example;

Thus 873,95 — 567,5 = 306,45 its Log. 2.486359
P = 567,5 its Logarith. is 2.753966 } Add
R = 0,06 its Logarith. is $\bar{8}$.778151 }
From the firſt Log. ſubſtract this Log. 1.532117
There remains the Logar. of 9 = *T* 0.954242

Which ſhews, that the Time ſought is juſt nine Years.

If the Work of theſe four Examples, and the Rules by which they are perform'd, be well underſtood, they will be ſufficient (*notwithſtanding there is really but one Queſtion, only it is varied according to the ſeveral Caſes*) to ſhew how any Queſtion of the like kind may be truly reſolv'd, at any propoſed Rate of Simple Intereſt, and for any aſſign'd Time; eſpecially if the Time given (or ſought) does conſiſt of compleat or whole Years.

But if the Time given (or ſought) does not conſiſt of whole Years, as moſt generally it does not, it being either leſs than a Year; or Years, and ſome Parts of a Year, as *Weeks*, *Months*, or *Quarters*, &c. then the odd Time, leſs than a compleat Year, muſt be reduced (or converted) into Decimal Parts of a Year (*per Sect.* 5. *p.* 7.) And unleſs ſuch Parts of a Year chance to be juſt $\frac{1}{4}$ $\frac{1}{2}$ or $\frac{3}{4}$ of a Year, then the beſt way will be to reduce the odd Time into Days, and then work with the Decimal Parts of a Year that are equivalent to thoſe Number of Days.

And for the eaſy and ready finding out of the true Number of Days that are contain'd between any two aſſigned Times leſs than a Year, and the Decimal Parts of a Year that are equal to thoſe Days, I have here inſerted two ſmall Tables.

The Uſe of the following Table of Months and Days, is no more but thus: Find the firſt propoſed Month at the top or head of its reſpective Part of the Table; and in the ſame Column under it, look for the other Month, and by it ſtands the Number of Days requir'd, according to the Title of the Table.

As for example; From the 1ſt, 5th, 10th, 17th, or any other Day in *April*, to the 1ſt, 5th, 10th, 17th, (viz. *to the ſame*) Day in *December*, is juſt 244 Days.

Or, From the 4th, 7th, or 12th, &c. of *October*, to the 4th, 7th, or 12th, &c. of *Auguſt*, is juſt 304 Days. And ſo the true Number of Days that are between any two of the ſame Days in the propoſed Months, may be found by Inſpection only.

A Table that ſhews, by Inſpection only, the true Number of Days, from every Day in any Month, to the ſame Day in any other Month, throughout the whole Year.

January	Februar.	March	April	May	June
Febr. 31	Mar. 28	Apr. 31	May 30	June 31	July 30
Mar. 59	Apr. 59	May 61	June 61	July 61	Aug. 61
Apr. 90	May 89	June 92	July 91	Aug. 92	Sept. 92
May 120	June 120	July 122	Aug. 122	Sep. 123	Oct. 122
June 151	July 150	Aug. 153	Sep. 153	Oct. 153	Nov. 153
July 181	Aug. 181	Sep. 184	Oct. 183	Nov. 184	Dec. 183
Aug. 212	Sep. 212	Oct. 214	Nov. 214	Dec. 214	Jan. 214
Sep. 243	Oct. 242	Nov. 245	Dec. 244	Jan. 245	Feb. 245
Oct. 273	Nov. 273	Dec. 275	Jan. 275	Feb. 276	Mar. 273
Nov. 304	Dec. 303	Jan. 306	Feb. 306	Mar. 304	Apr. 304
Dec. 334	Jan. 334	Feb. 337	Mar. 334	Apr. 335	May 334
Jan. 365	Feb. 365	Mar. 365	Apr. 365	May 365	June 365

July	Auguſt	Septem.	Octob.	Novem.	Decem.
Aug. 31	Sept. 31	Oct. 30	Nov. 31	Dec. 30	Jan. 31
Sept. 62	Oct. 61	Nov. 61	Dec. 61	Jan. 61	Feb. 62
Oct. 92	Nov. 92	Dec. 91	Jan. 92	Feb. 92	Mar. 90
Nov. 123	Dec. 122	Jan. 122	Feb. 123	Mar. 120	Apr. 121
Dec. 153	Jan. 153	Feb. 153	Mar. 151	Apr. 151	May 151
Jan. 184	Feb. 184	Mar. 181	Apr. 182	May 181	June 182
Feb. 215	Mar. 212	Apr. 212	May 212	June 212	July 212
Mar. 243	Apr. 243	May 242	June 243	July 242	Aug. 243
Apr. 274	May 273	June 273	July 273	Aug. 273	Sept. 274
May 304	June 304	July 303	Aug. 304	Sept. 304	Oct. 304
June 335	July 334	Aug. 334	Sept. 335	Oct. 334	Nov. 335
July 365	Aug. 365	Sept. 365	Oct. 365	Nov. 365	Dec. 365

But if the two given Days of the Month are different, then their Difference muſt be added to the Number found in the Table. As, ſuppoſe, between the 4th of *October* and the 25th of *Auguſt:* Here, becauſe the 25th of the one Month is 21 Days more than the 4th Day of the other Month, therefore the Number of Days requir'd will be 304 + 21 ; *viz.* 325 Days; and ſo for any other two Months and Number of Days in any propoſed Time throughout the Year; as in the Table above.

A Table for the ready finding the Decimal Parts of a Year, equal to any Number of Days, &c.

Days.	*Dec.Parts*	*Days.*	*Dec.Parts*	*Days.*	*Dec.Parts*
1 =	0,002740	10 =	0,027397	100 =	0,273973
2 =	0,005479	20 =	0,054794	200 =	0,547945
3 =	0,008219	30 =	0,082192	300 =	0,821918
4 =	0,010959	40 =	0,109589	365 =	1,000000
5 =	0,013699	50 =	0,136986		
6 =	0,016438	60 =	0,164383	¼ *of a Year* =	0,25
7 =	0,019178	70 =	0,191781	½ *a Year* =	0,5
8 =	0,021918	80 =	0,219178	¾ *of a Year* =	0,75
9 =	0,024657	90 =	0,246575		

The Uſe of this Table is thus;

If the propoſed Number of Days can be exactly found in the Table (under *Days*) their Decimal Parts are alſo found againſt them by Inſpection only.

But if the true Number of Days cannot be exactly found there, then both they, and their Decimal Parts muſt be collected out of the Table at twice, or thrice, according as their Number requires.

As for Example: Suppoſe it were required to find the Decimal Parts of a Year equal to 135 Days?

Days. *Dec.Parts.*

Then { 100 = 0,273973
30 = 0,082192
5 = 0,013699 } Add all theſe together.

Hence 135 = 0,369864 the Decimal Parts required.

And thus may the Decimal Parts of a Year, equivalent to any given Number of Days, be very eaſily found to ſix Places of Figures; but for common Buſineſs, it may ſuffice to work with only four of thoſe Places; and in ſmall Sums, two or three Places (according to Diſcretion) may be near enough to the Truth.

Or the Decimal Parts, equivalent to any given Number of Days, may be found by the Logarithms,

Thus, { *From the Log. of the given Number of Days, ſubſtract the Log. of 365 Days, (viz. 2,562293) and there will remain the Log. of the Decimal Parts equal to thoſe Number of Days.*

Example.

Let it be required to find the Decimal Parts of a Year equal to 135 Days; as before.

$$\left.\begin{array}{l}\text{Here is given } 135 \text{ its Logar. is } 2.130334 \\ \text{Days in a Year } 365 \text{ its Logar. is } \underline{2.562293}\end{array}\right\}\text{Subſtract}$$

And there remains the Log. $\overline{9}.568041$ of 0,369863 the Decimal Parts equal to 135 Days, &c. as above.

Theſe things being underſtood, it will be as eaſy to calculate any Queſtion relating to any of the foregoing Caſes, when the Time given, or ſought, is either leſs than a Year, or Years and Parts of a Year; as it is for thoſe in whole Years only.

As for inſtance in *Caſe* 1. Suppoſe it were required to find *What Sum* 780 l. 15 s. *would amount to in* 3 *Years and* 179 *Days, at Five and a half* per Cent?

In this Queſt. there is given $P = 780,75$ $R = 0,055$ and $T = 3,4904$ found by the Table.

$$\left.\begin{array}{rl}\text{Thus, Three Years} = & 3.000000 \\ \text{Days}\left\{\begin{array}{r}100 \\ 70 \\ 9\end{array}\right. \begin{array}{l}= \\ = \\ =\end{array} & \begin{array}{l}0.273973 \\ 0.191781 \\ \underline{0.024657}\end{array}\end{array}\right\}\text{Add}$$

Hence 3 Years and 179 Days is $= 3,490411 = T$.

$$\left.\begin{array}{lll}\text{Then } P = 780,75 & \text{its Log.} & 2.892512 \\ T = 3,4904 & \text{its Log.} & 0.542875 \\ R = 0,055 & \text{its Log.} & \underline{\overline{8}.740363}\end{array}\right\}\text{Add}$$

The Sum is the Logarithm 2.175750 of 149,88

That is, 149,88 = 149 *l.* 17 *s.* 7 *d.* the Intereſt. To which
Add the Principal 780 *l.* 15 *s.* 0 *d.*
And the Sum is 930 *l.* 12 *s.* 7 *d.* the *Amount* required.

Again, That I may make all as plain as I can, take an *Example* in *Caſe* 4. *viz.* to find the Time.

Let it be required to find, *In what time* 780 l. 15 s. *would amount to the Sum of* 930 l. 12 s, and 7 d. *at the Rate of* 5 ½ per Cent. per Annum?

In this Example there is given A=930,63 R=0,055 and P= 780,75 to find T: Which is done thus:

930,63 — 780,75 = 149,88 its Log. 2.175750
And P= 780,75 its Log. 2.892512 } Add
R=0,055 its Log. 8.740363 }
From the firſt Log. ſubſt. this Log. 1.632875
And there remains the Log. 0.542875 of 3.4904

That is, 3 Years, and 0,4904 Decimal Parts of a Year; but 0,4904 = 179 Days; conſequently 3 Years and 179 Days is the time required by the Queſtion.

Or the Decimal Parts of a Year may be otherwiſe reduced into Days, by help of the laſt Table:

As in this example: From 0,4904 the given Parts,
take the next leſs tabular N°. 0,2740 = 100 Days.
There remains 0,2164
Again, From the Rem. take 0,1918 = 70 Days.
Laſtly this 0,0246 = 9 Days.

So that 0,4904 = 179 Days, by the Table, &c, as before.

Or the Decimal Parts of a Year may be reduced into Days by the Logarithms;

Thus, { *To the Log. of the propoſed Parts, add the Log. of 365, the Days in a common Year, and the Sum will be the Log. of the Days required.*

For inſtance, in the laſt Example, wherein the given
Decimal Parts are 0,4904 Log. 9.690550 } Add
The Days in a Year are 365 its Log. 2.562293 }
The Sum is this Logarithm 2.252843 of 179
&c.

Sect.

Sect. 2. *Of* Annuities, *or* Penſions, *&c. in Arrears, computed at Simple Intereſt.*

I Shall here make uſe of the ſame Letters to denote the ſeveral parts of the Queſtion, as before, ſave only *U*.

U. { Denotes the Annuity or Penſion, *&c. viz.* either yearly, half yearly, or quarterly Rents.

T. { The Time of its being unpaid, *viz.* the Number of all the Payments that are in Arrears.

A. { The Amount of the Annuity and its Intereſt; *viz.* the Sum of all the Arrears due.

R. { The Ratio of the Rate, *viz.* the Intereſt of 1 *l.* as before.

Then if $U=$ the firſt Year's Rent without its Intereſt;

$$\text{And}\left.\begin{array}{l} RU=\text{the Intereſt} \\ 2U=\text{the Rent}\end{array}\right\}\text{due the ſecond Year.}$$

$$\text{And}\left.\begin{array}{l} 2RU=\text{the Intereſt} \\ 3U=\text{the Rent}\end{array}\right\}\text{due the third Year.}$$

$$\text{And}\left.\begin{array}{l} 3RU=\text{the Intereſt} \\ 4U=\text{the Rent,}\end{array}\right\}\text{due the 4th Year.}$$

&c.

Hence it is plain, that $RU+2RU+3R+4U=A$, the Sum of all the Rents, and their Intereſts being forborn 4 Years. Here $T=4$, ſo that $RU+2RU+3RU=A-TU$. This Series being truly manag'd.

$$\text{Then will}\left\{\frac{TTR-TR}{2}=\frac{A-TU}{U}.\right.$$

From this Equation is deduced the following Rules, which admits of four Caſes.

Note, *I do here take it for granted, that the* Reader *doth by this time very well know how to perform Multiplication and Diviſion of any given Numbers, by Logarithms; (as directed in* Chap. I. *and practiſed in all the Examples of the laſt* Section:) *And therefore I ſhall, for brevity ſake, omit ſetting down in Words at length (as in the laſt* Section) *how the following Caſes are to be reſolved by Logarithms; that Work being* *ſo eaſily underſtood, by having a due regard to their reſpective Rules; that, it ſeems to me, to be wholly needleſs to repeat in Words how it is to be done: but only to ſet down the Work at large of all the Examples, according to the import of their reſpective Rules, by which they are computed with the Pen only.*

Caſe 1. Having *U*, *T*, and *R*, given; to find *A*: That is, if any Annuity, or Rent, with the time of its being unpaid, and the Rate of Intereſt *per Cent.* be given; thence to find what Sum all thoſe Arrears will amount to in that time, allowing any aſſigned Rate of Simple Intereſt for every particular Payment, as it becomes due.

Rule. *Multiply the Time, the Time leſt* 1, *and half the Ratio of the Rate, all three together, and to their Product add the Time; then multiply that Sum with the Annuity, and the Product will ſhew the Amount required.*

Example.

Suppoſe 356 *l.* Annuity or yearly Rent, be forborn or unpaid nine Years; *What Sum will all thoſe Arrears amount to in that time, allowing* 6 per Cent. per Ann. *for each Payment as it becomes due?*

In this example there is given $U=356$, $T=9$, and $R=0{,}06$; to find *A*.

First by the *Pen* only.

Let the Work be prepared thus, $9-1=8$, and $\frac{1}{2}R=0{,}03$

Then $9\times 8=72$; and $72\times 0{,}03=2{,}16$; to which adding the Time, it will be $2{,}16+9=11{,}16$; and $11{,}16$ multiplied with 356 is $3972{,}96=A$.

That is, 3972 *l.* 19 *s.* $2\frac{1}{2}$ *d.* will be the Amount, as was required.

The ſame perform'd by *Logarithms.*

First $T=9$ its Logar. is 0.954242 } Add
And $T-1=8$ its Logar. is 0.903090 } Add
Again, * $R=0{,}03$ its Logar. is $\bar{8}.477121$ } Add
Their Sum is the Log. 0.334453 of 2.16

$$\left.\begin{array}{ll}\text{Then } 2{,}16 + 9 = 11{,}16 \text{ its Log. is} & 1.047664 \\ \text{And } U = 356 \text{ its Log. is} & 2.551450\end{array}\right\}\text{Add}$$

The Sum is the Logarithm 3.599114 of 3972,96
That is 3972 *l.* 19 *s.* 2½ *d.* = A, the Amount or Sum of all the Arrears, as before by the Pen.

Caſe 2. When A, T, and R, are given, to find U. That is, to find what Annuity, or Yearly Rent, being forborn or unpaid any aſſign'd Time, will amount to a propoſed Sum, allowing any given Rate of Intereſt *per Cent.* for every Payment as it becomes due.

Rule. *Multiply the Time, the Time leſs* 1, *and half the Ratio of the Rate, all three together; and to their Product, add the Time (as before:) Then divide the propoſed Amount by that Sum, and the Quotient will ſhew the Annuity, or yearly Rent required.*

Example.

Suppoſe it were required to find, *What Annuity, or yearly Rent will amount to (or raiſe a Stock of)* 3972 *l.* 19 s. 2½ d. *allowing* 6 per Cent. *for every Payment as it becomes due?*

In this Queſtion there is given $A = 3972{,}96$; $T = 9$; and $R = 0{,}06$; to find U: which may be thus done.

Firſt by the Pen only; Thus,

The Work prepared is $9 - 1 = 8$, and $\frac{1}{2}R = 0{,}03$; then $9 \times 8 = 72$, and $72 \times 0{,}03 = 2{,}16$; which being added to the Time, is $9 + 2{,}16 = 11{,}16$ for a Diviſor.

Then 11,16) 3972,96 (356 = U

Viz. 356 *l.* will be the Yearly Rent required.

The ſame perform'd by Logarithms.

$$\left.\begin{array}{lll}\text{Thus, } T = 9 & \text{its Log. is} & 0.954242 \\ T - 1 = 8 & \text{its Log. is} & 0.903090 \\ \text{And } \frac{1}{2}R = 0{,}03 & \text{its Log. is} & 8.477121\end{array}\right\}\text{Add}$$

The Sum is the Logarithm 0.334453 of 2,16

$$\left.\begin{array}{ll}\text{Again, } A = 3972{,}96 \text{ its Log. is} & 3.599113 \\ \text{And } 9 + 2{,}16 = 11{,}16 \text{ its Log.} & 1.047664\end{array}\right\}\text{Subſtract}$$

There remains the Logarithm 2.551449 of 356 = U

That is, 356 *l.* is the Annuity, or yearly Rent, as before.

Caſe 3. When U, A, and R, are given; to find T. That is, to find the Time in which any Annuity, or yearly Rent, being forborn or unpaid, will amount to any propoſed Sum; allowing any given Rate of Intereſt *per cent.* &c. for each Payment, as it becomes due.

Rule. *Subſtract the Ratio of the Rate from* 2; *then divide the Remainder by twice the Ratio, and call the Quotient x.*
Next, Square that Quotient (*viz.* multiply it with itſelf) *and call that Square xx.*
Then divide twice the propoſed Amount by the Product of the Annuity multiplied with the Ratio, and to the Quotient, add the Square Number called xx: Then extract the Square Root of that Sum, and from that Root ſubſtract the Number called x, and the Remainder will ſhew the Time ſought.

Example.

What Time will 356 l. *yearly Rent require to raiſe the Sum of* 3972 l. 19 s. $2\frac{1}{2}$ d. *at Six* per Cent. &c. *for each Payment, as it becomes due?*

In this Queſtion there is given, $U = 356$; $R = 0,06$; and $A = 3972,96$; to find T.

And firſt by the Pen only:

Firſt, $2 - 0,06 = 1,94$ and $2R = 0,12)\ 1,94(16,166 = x$
Next, $16,166 \times 16,166 = 261,361$ the Number call'd xx.
Then $2A = 7945,92$ and $356 \times 0,06 = 21,36$
And $21,36)\ 7945,92\ (372,003$
Then $372,003 + 261,361 = 633,364$. whoſe Square Root is 25,166. Laſtly, $25,166 - 16,166$ leaves $9 = T$; *viz.* 9 Years, is the Time required by the Queſtion.

The ſame found by Logarithms.

Firſt $2 - 0,06 = 1,94$ its Log. 0.287802 } Subſtract
And $2R = 0,12$ its Logar. is 9.079181 }
There remains the Logar. 1.208621 of 16,166 call'd x.
Multiplied with 2
The Product is the Logarithm 2.417242 of 261,361 call'd xx

Again; 2 $A = 7945,92$ its Log. 3.900145

And $U = 356$ its Log. is 2.551449 } Add
$R = 0,06$ its Log. is 8.778151 }

From the firſt Log. ſubſtr. this Log. 1.329600

And there remains the Logar. 2.570545 of 372,003
Then $372,003 + 261,361 = 633,264$ its Log. 2.801585
The half of that Log. is 1.400792

Laſtly, The Number which belongs to the half Logarithm is 25,166 From which ſubſtract the Number called $x = 16,166$

Remains $9,000 = T$; the Numb. of Years ſought, *&c.*

Caſe 4. If U, T, and A, are given; to find R. That is, Having any Annuity, or yearly Rent, with the Time of its being unpaid, and the Sum it's propoſed to amount to in that Time given; thence to find, what Rate of Intereſt *per Cent.* muſt be allow'd for every Payment as it becomes due.

Rule. { *Multiply the Annuity with the Time, and multiply that Product with the Time again; and make half the Difference of thoſe two Products a Diviſor.*
Next, Subſtract the firſt Product from the propoſed Amount, then divide the Remainder by the aforeſaid Diviſor, and the Quotient will be the Ratio of the Rate of the Intereſt required.

Example.

Suppoſe 356 l. *yearly Rent, being forborn or unpaid nine Years, be propoſed to raiſe the Sum of* 3972 l. 19 s. 2 ½ d. *what Rate of Intereſt* per Cent. *muſt be allow'd for every Payment as it becomes due?*

In this Queſtion there is given $U = 356$; $T = 9$; and $A = 3972,96$; to find R. Which may be thus found.

Firſt by the Pen only;

Thus, $356 \times 9 = 3204$; and $3204 \times 9 = 28836$

Next, $28836 - 3204 = 25632$ its half is 12816 for the Diviſor.

And $3972,96 - 3204 = 768,96$ for the Dividend.

Then 12816) 768,96 (0,06 = R the Ratio.

And it will be, As 1 : is to 0,06 :: ſo is 100 : to 6 the Rate of Intereſt *per Cent.* &c, as was required.

The ſame may be done by Logarithms.

Thus $U = 356$ its Log. 2.551450 } Add
And $T = 9$ its Log. 0.954242 }
To the laſt Log. add this 3.505692 its Numb. is 3204
Their Sum 4.459934 its Num. is 28836
The Difference of the two Numbers is 25632
and the half of 25632 is 12816 for the Diviſor.

From $A = 3972,96$ take the firſt Numb. 3204, and there remains 768,96 its Log. 2,885904 } Subſtract
and 12816 the Diviſ. its Log. 4.107752 }

There remains the Logar. 8.778152 of 0,06 $= R$,
Then 1 : 0,06 :: 100 : 6 the Rate of Intereſt *per Cent.* &c.

Thus you have all four Caſes relating to *Annuities*, or *Rents*, &c. in Arrears, with their Examples in Yearly Payments: But if the *Annuities*, or *Rents*, are to be paid by Half-yearly or Quarterly Payments, as moſt generally they are,

Then,

1. Inſtead of the Ratio of the given Rate of Intereſt, you muſt take the ½ of that Ratio for Half-yearly Payments, and the ¼ of it for Quarterly Payments, *&c.*

2. And you muſt take the ½ of the Yearly Rent for Half-yearly Payments, and the ¼ of it for Quarterly Payments, *&c.*

3. But inſtead of the propoſed Number of Years, you muſt take twice that Number for Half-yearly Payments, and four times that Number for Quarterly Payments, *&c.* as in the following Examples,

Examples in Half-yearly Payments.

Suppoſe 356 l. per Annum *Annuity, payable every Half-year, were forborn or unpaid nine Years; what would all thoſe Arrears amount to, at the Rate of* 6 per Cent. per Annum? &c.

In this Queſtion there is given $U = 178$, *viz.* the ½ of 356*l.* $R = 0,03$ the ½ of the Ratio of 6 *per Cent.* and $T = 18$; *viz.* 9 × 2 the Number of Half-years in nine Years; thence to find A. (*per Rule at Caſe* 1.).

First to prepare the Work $18 - 1 = 17$;
and $\frac{1}{2} R = 0,015$.

Then $18 \times 17 = 306$; and $306 \times 0{,}015 = 4{,}59$; to which add the Time, *viz.* $4{,}59 + 18 = 22{,}59$. And then $22{,}95 \times 178$ gives $4021{,}02 = A$, the Amount or Sum of all the Arrears, as was required.

The ſame Examples in Quarterly Payments.

That is, 356 l. *a Year, to be paid every Quarter, being forborn nine Years, what Sum will it amount to, allowing* 6 per Cent, &c. *for every Payment as it becomes due?*

Now here it will be $U = 89$, *viz.* the ¼ of 365 *l.* $R = 0{,}015$, *viz.* the ¼ of 0,06 the Ratio of the given Rate *per cent.* &c. and $T = 36$, *viz.* 9×4 the Number of Quarters in nine Years: thence to find A, as before.

To prepare the Work $36 - 1 = 35$,
and $\frac{1}{2} R = 0{,}0075$.

Then $36 \times 35 = 1260$; and $1260 \times 0{,}0075 = 9{,}45$; to which add the Time, *viz.* $9{,}45 + 36 = 45{,}45$, and then $45{,}45 \times 89$ gives $4045{,}05 = A$, the Amount or Sum of all the Arrears at Quarterly Payments.

By comparing the Work of theſe two Examples with that in *pag.* 144. it may be obſerved, that half-yearly Payments are more advantageous than yearly Payments; and Quarterly are yet more advantageous than half-yearly.

For yearly Payments amount but to 3972 *l.* 19 *s.* 2½ *d.*
But ½ yearly Payments amount to 4021 *l.* 00 *s.* 3½ *d.*
And quarterly Payments amount to 4045 *l.* 01 *s.* 00 *d.*

If the two laſt Examples be well conſider'd and underſtood, it will be eaſy to conceive how to compute any Queſtion in the other three Caſes, when the Payments are either half-yearly, or quarterly; and therefore I ſhall omit inſerting Examples, and proceed to the next Section.

Sect. 3. *The Preſent Worth of Annuities and Penſions, &c. computed at Simple Intereſt.*

IN this Section I ſhall make uſe of theſe Letters to denote the ſeveral Parts of the Queſtion:

Viz. U denotes the *Annuity* or *Rent.*
T the *Time* of its *Continuance.*
R the *Ratio* of the *Rate* of *Intereſt.*
as before.

And P denotes the *Preſent Worth* of the *Annuity.*

Then if the General Rules or Theorems of the two laſt Sections be combined into one: that is, in the firſt it is $TRP + P = A$. vid. *Sect.* 1.

In the ſecond $\left\{ \frac{TTR - TR}{2} = \frac{A - TU}{U} \right.$

That is, $TTRU - TRU + 2TU = 2A$

And $2TRP + 2P = 2A$. Which gives this

Theorem, $TTRU - TRU + 2TU = 2TRP - 2P$

And from this Theorem is deduced the following Rules, which alſo admit of four Caſes, as in the laſt Section.

Caſe 1. Having U, T, and R given; to find P.

That is, having any Annuity or yearly Rent, and the Time of its Continuance given; to find the preſent Worth of that Annuity, or Leaſe, &c. allowing any given Rate of Simple Intereſt *per cent.* to the Purchaſer for his ready Money.

Rule. *Multiply the Time, the Time leſs* 1, *and half the Ratio of the Rate, all three together, and to their Product add the Time; then multiply that Sum with the Annuity.* (So far the Work is juſt the ſame as in the laſt Section.) *Then divide that Product, by the Product of the Ratio into the Time, added to* 1; *and the Quotient will ſhew the preſent Worth required.*

Example.

Suppoſe a Leaſe of 250 l. per Annum, *the Rents to be paid half-yearly, were to be lett for* 21 *Years; what may the preſent Worth of that Leaſe be, at the Rate of* 5 per Cent. &c?

In this Queſtion there is given $U = 125$, *viz.* the $\frac{1}{2}$ of 250; $R = 0{,}025$ the $\frac{1}{2}$ of 0,05 the Ratio of 5 *per cent.* and $T = 42$, (*viz.* 21×2) the Number of Half-years in 21 Years; to find P.

Firſt, the Work prepared ſtands thus, $42 - 1 = 41$, and $\frac{1}{2}R = 0{,}0125$.

Then $42 \times 41 = 1722$; and $1722 \times 0{,}0125 = 21{,}525$

Next, $21{,}525 + 42 = 63{,}525$; which multiplied with 125, the half-yearly Payment, is 7940,625 for the Dividend.

Again, $42 \times 0{,}025 = 1{,}05$; and $1{,}05 + 1 = 2{,}05$ for the Diviſor.

Then $2{,}05)\, 7940{,}625\, (3873{,}4756 = P$.

Viz. 3873 *l.* 9 *s.* 6 *d.* will be the preſent Worth of ſuch a Leaſe as was required.

The ſame perform'd by Logarithms:

First $T = 42$ its Log. is 1.623249 }
And $T - 1 = 41$ its Log. is 1.612784 } Add
$\frac{1}{2} R = 0{,}0125$ its Log. is $\bar{8}$.096910 }
The Sum is the Logarith. 1.332943 of 21,525
And $21{,}525 + 42 = 63{,}525$ its Log. 1.802945 } Add
$U = 125$ its Log. is 2.096910 }
The Sum is the Log. of the Dividend 3.899855
Again, $T = 42$ its Logar. is 1.623249 } Add
And $R = 0{,}025$ its Log. is $\bar{8}$.397940 }
The Sum is the Logarithm 0.021189 of 1,05
And $1{,}05 + 1 = 2{,}05$ for the Diviſor.
Then the Log. of the Divid. is 3.899855 } Subſtract
And the Log. of 2,05 the Diviſ. 0.311754 }
There remains the Log. 3.588101 of 387347

That is, 3873 *l.* 9 *s.* 4¾ *d.* is here found to be the preſent Worth of that Annuity; as before, *fere.*

Caſe 2. When *P*, *T*, and *R*, are given; to find *U*. *Viz.* To find what Annuity, or Leaſe, &c. may be purchaſed for any propoſed Sum, to continue any aſſigned Time, and at any given Rate of Intereſt.

Rule. { *Multiply the Time, the Ratio of the Rate, and the preſent Worth,* (*viz.* the Purchaſe-Money) *all three together; and to their Product add the preſent Worth; make that Sum a Dividend. Then multiply the Time, the Time leſs* 1, *and half the Ratio together; and to their Product add the Time; make that Sum a Diviſor; the Quotient ariſing from thence, will ſhew the Annuity, or Rent required.*

Example.

What Annuity, or Rent, to be paid half-yearly, and to continue 21 *Years, may be purchaſed for the Sum of* 3873 l. 9 s. 6 d. *at the Rate of* 5 per Cent?

In this Example there is given $P = 3873{,}475$; $T = 42$, *viz.* 21×2, the Half-years in 21 Years, and $R = 0{,}25$ the $\frac{1}{2}$ of the Ratio $0{,}05$ of 5 *per cent*; to find U, the half-yearly Rent.

The Work prepared is $42 - 1 = 41$; and $\frac{1}{2} R = 0{,}0125$

Then $3873{,}475 \times 42 = 162685{,}95$ and $162685{,}95 \times 0{,}025$ gives $4067{,}14875$; and $4067{,}14875 + 3873{,}475 = 7940{,}62375$ for the Dividend.

Again, $42 \times 41 = 1722$ and $1722 \times 0{,}0125 = 21{,}525$ and $21{,}525 + 42 = 63{,}525$ for the Diviſor.

Then $63{,}525)\,7940{,}62375\,(125 = U$, the half-yearly Rent, conſequently $125 \times 2 = 350\,l.$ will be the Annuity or Rent *per Annum*, as was required.

The ſame done by Logarithms.

$$\left.\begin{array}{r}\text{Firſt } P = 3873{,}475 \text{ its Log. is } 3.588101\\ \text{And } T = 42 \text{ its Log. is } 1.623249\\ R = 0{,}0125 \text{ its Log. is } \underline{\overline{8}.397940}\end{array}\right\}\text{Add}$$

The Sum is the Logarithm 3.609290 of 4067,15

And $4067{,}15 + 3873{,}475 = 7940{,}625$ the Dividend.

$$\left.\begin{array}{r}\text{Again, } T = 42 \text{ its Log. is } 1.623249\\ T - 1 = 41 \text{ its Log. is } 1.612784\\ \text{And } \frac{1}{2} R = 0{,}0125 \text{ its Log. is } \underline{\overline{8}.096910}\end{array}\right\}\text{Add}$$

The Sum is the Logarithm 1.332943 of 21,525

And $21{,}525 + 42 = 63{,}525$ for the Diviſor.

$$\left.\begin{array}{r}\text{Then the Divid. } 7940{,}625 \text{ its Log. } 3.899854\\ \text{The Diviſor } 63{,}525 \text{ its Log. is } \underline{1.802945}\end{array}\right\}\text{Subſtract}$$

There remains the Logar. 2.096909 of 125

Viz. 125 *l.* is the half-yearly Rent, &c. as before.

Caſe 3. When U, P, and R, are given; to find T. That is, when any Annuity, with its propoſed Value or Purchaſe, and the Rate of Intereſt *per cent.* are given; thence to find how long the Purchaſer ought to enjoy it.

Rule. { *To the preſent Worth, add half the Annuity; then multiply that Sum with the Ratio, and ſubſtract the Product from the Annuity; then divide the Remainder by the Product of the Annuity multiplied with the Ratio, and call the Quotient x.*

Next,

Rule. *Next, Square that Quotient* (viz. *multiply it with itſelf*) *and call that Square xx.*
Then divide the preſent Worth by the ſame Diviſor, viz. *by the Product of the Annuity into the Ratio; and to twice that Quotient add the Square Number called xx: then extract the Square Root of their Sum, and from that Root ſubſtract the Number called x; the Remainder will ſhew the Time ſought.*

Example.

Suppoſe one would lay out 3873 l. 9 s. 6 d. *ready Money for an Annuity of* 250 l. per Annum, *to be paid by half-yearly Payments; how long may he enjoy that Annuity, to be allow'd but* 5 per Cent. per Annum, *Simple Intereſt for his Money?*

Or thus; which is the ſame thing;

In what Time will 250 l. per Annum *pay off a Debt of* 3873 l. 9 s. 6 d. *by half-yearly Payments, allowing the Creditor* 5 per Cent. *Intereſt for his Money, until the Debt be diſcharged?*

In this Queſtion there is given $P = 3873,475$; $U = 125$; *viz.* the half-yearly Payments; and $R = 0,025$, *viz.* the ½ of the Ratio of 5 *per cent.* thence to find T, the Number of Half-Years.

Firſt $\frac{1}{2}U = 62,5$ and $62,5 + 3873,475 = 3935,975$
Then $3935,975 \times 0,025 = 98,4$ *fere*, and $125 - 98,4$
Leaves 26,6 for the firſt Dividend.

And $125 \times 0,025 = 3,125$ for the common Diviſor.
Then 3,125) 26,600 (8,512 the Number call'd *x*.
And $8,512 \times 8,512 = 72,454$ the Number call'd *xx*.

Again, the common Diviſor is 3,125

And 3,125) 3873,475 (1239,512

$$\left.\begin{array}{l}\text{And twice } 1239,512 \text{ is } 2479,024\\ \text{The Number call'd } xx \text{ is } \underline{\quad 72,454}\end{array}\right\}\text{Add}$$

And the Square Root of 2551,478 is 50,512

Laſtly, $50,512 - 8,512$ (*the Number call'd x*) leaves $42 = T$ the Number of Half-Years; conſequently the Half of 42, *viz.* 21 will be the Time or Number of Years required by the Queſtion.

This Caſe may alſo be reſolved by Logarithms, but it is attended with ſome ſeeming Difficulties, which require a due Conſideration of the Rule.

As for inſtance in the laſt Example, wherein $P = 3873{,}475$; $U = 125$; and $R = 0{,}025$; to find T.

$$\left.\begin{array}{r}62{,}5 + 3873{,}475 = 3935{,}975 \text{ its Log. } 3.595053 \\ R = 0{,}025 \text{ its Log. is } 8.397940\end{array}\right\}\text{Add}$$

The Sum is the Log. of 98,4 — 1.992993

125 — 98,4 = 26,6 its Log. 1.424882 of the Divid.

$$\left.\begin{array}{r}\text{Again, } U = 125 \text{ its Log. is } 2.096910 \\ \text{And } R = 0{,}025 \text{ its Log. is } 8.397940\end{array}\right\}\text{Add}$$

The Sum is the Log. of 3,125 : 0.494850 the Diviſor.

To be ſubſt. from the upper Log. 0.930032 Remains the Logarithm of 8,512 the Number call'd x.

And the double of the laſt Log. is 1.860064 which is the Logarithm of 72,454 the Number call'd xx.

$$\left.\begin{array}{r}\text{Again, } P = 3873{,}475 \text{ its Log. } 3.588100 \\ \text{And the Log. of the Diviſor is } 0.494850\end{array}\right\}\text{Subſtract}$$

There remains the Logarithm 3.093250 of 1239,512

$$\left.\begin{array}{r}\text{Twice } 1239{,}512 \text{ is } 2479{,}024 \\ \text{The Number call'd } xx \quad 72{,}454\end{array}\right\}\text{Add}$$

The Sum is 2551,478 its Log. is 3.406792

The Half of the laſt Log. is 1.703396 its Number is 50,512 from which ſubſtract the Number call'd $x =$ 8,512

Remains 42,000 $= T$ the Number of Half-years, &c. as before.

Caſe 4. When U, P, and T, are given; to find R.

That is, when any Annuity, with the Time of its Continuance, and its propoſed Preſent Worth for that Time are given; thence to find what Rate of Intereſt *per cent.* is allow'd the Purchaſer.

Rule. *Multiply the Annuity with the Time, and from the Product ſubſtract the preſent Worth; then make twice that Remainder a Dividend.*

Next, Add the Annuity to twice the preſent Worth, and from their Sum ſubſtract the firſt Product; then multiply the Remainder with the

the Time, and make the Product a Diviſor; by which divide the aforeſaid Dividend, and the Quotient will ſhew the Ratio of the Rate of Intereſt required.

Example.

Suppoſe 3873 l. 9 s. 6 d. *were given for an Annuity of* 250 l. per Annum, *to be paid half-yearly, and to continue* 21 *Years; what Rate of Intereſt* per cent. &c. *is allow'd the Purchaſer.*

Here is given $P = 3873{,}475$; $U = 125$ the half-yearly Payments; and $T = 42$, the Number of Half-years in 21 Years.

Then $125 \times 42 = 5250$ the firſt Product;
And $5250 - 3873{,}475 = 1376{,}525$ its Double is 2753,05 for the Dividend.

Next, $2P = 7746{,}95$ and $7746{,}95 + 125 = 7871{,}95$ and $7871{,}95 - 5250$ (*the firſt Product*) leaves 2621,95

Again, $2621{,}95 \times 42 = 110121{,}9$ for a Diviſor.

Then 110121,9) 2753,05 (0,025 $= R$ the Ratio of the Rate *per cent.* for half-yearly Payments, conſequently $0{,}025 \times 2 = 0{,}05$ is the Ratio of the Rate *per cent. per annum*: And then it will be,

As 1 : to 0,05 : : ſo is 100 : to 5, the Rate of Intereſt as was required.

The ſame may be done by Logarithms,

$$\left.\begin{array}{ll}\text{Thus, } U = 125 \text{ its Log. is} & 2.096910 \\ \text{And } T = 42 \text{ its Logar. is} & \underline{1.623249}\end{array}\right\}\text{Add}$$

The Sum is the Logarithm 3.720159 of 5250

And $5250 - 3873{,}475 = 1376{,}525$ the Double whereof is 2753,05 its Log. is 3.439814 *to be reſerv'd.*

Next, $2P = 7746{,}95$ and $7746{,}95 + 125 = 7871{,}95$ and $7871{,}95 - 5250$ the firſt Product or Number above.

$$\left.\begin{array}{ll}\text{There remains } 2621{,}95 \text{ its Log. is} & 3.418624 \\ \text{The Time } 42 = T\text{, its Logar. is} & \underline{1.623249}\end{array}\right\}\text{Add}$$

Subſtract this Logarithm 5.041873 Sum
From the Reſerved Logar. *viz.* 3.439814

And there remains the Logarith. $\overline{8}.397941$ of 0,025

Viz. $0{,}025 = R$ the Ratio of the Rate *per cent.* for half-yearly Payments, *&c.* as above.

Theſe four Examples may be ſufficient to ſhew how any other of the like kind may be reſolved, either for whole Years, or thoſe of Quarterly Rents, *&c.* provided the purchaſed Rent, Leaſe, or Annuity is to commence immediately.

But if it be required to find the preſent Value, or Worth of any Rent, or Annuity, *&c.* in Reverſion; that is, when it is not to be enter'd upon until after ſome Time or Number of Years are paſt;

Then, *You muſt firſt find what the propoſed Annuity, or Rent,* &c. *would be worth for the given Time of its Continuance, as if it were to be immediately enter'd upon:*
And then you muſt find what Principal or Sum being forborn at Intereſt during the Time of that Reverſion, would amount to or raiſe the aforeſaid Value; that Principal will be the Sum which ſhould be paid for the propoſed Annuity in Reverſion.

Example.

There is the Reverſion of a Leaſe of 175 l. per Annum, *to be lett for eleven Years, which are to commence after nine Years are expired; 'tis required to find the preſent Worth of that Leaſe, allowing the Purchaſer* 6 per Cent. *for his ready Money.*

The firſt Work in this Queſtion muſt be to find what 175 *l. per Annum*, to continue eleven Years, is worth in ready Money, ſuppoſing it were to be immediately enter'd upon: as in *Caſe* 1. *pag.* 150.

That is, here is given $U = 175$; $T = 11$; and $R = 0,06$ to find P, the preſent Worth.

The Work prepared will ſtand thus $11 - 1 = 10$ and $\frac{1}{2} R = 0,03$. Then $11 \times 10 \times 0,03 = 3,3$ and $3,3 + 11 = 14,3$.

Next, $14,3 \times 175 = 2502,5$ for a Dividend.

Again, $11 \times 0,06 = 0,66$ and $0,66 + 1 = 1,66$ the Diviſ.
Then $1,66) 2502,5 (1507,53 = P$ the preſent Worth, if the Leaſe were to commence immediately; but becauſe it is not ſo, therefore

The next Work muſt be to find what Principal or Sum, being put out to Intereſt for nine Years, at 6 *per cent*, *&c.*

will amount to 1507,53 *l.* the which will be found (*per Caſe* 2. *pag.* 136.) to be 978,9162 = 978 *l.* 18 *s.* 3$\frac{3}{4}$ *d.* which is the true preſent Worth of that Leaſe in Reverſion, as was required.

2. Or if it be required to find what Annuity, *&c.* in Reverſion, may be purchaſed for any propoſed Sum, and at any given Rate of Intereſt *per cent.* when the Time the Annuity is not to be enter'd upon, and the Time of its Continuance, are both given:

Then { *The firſt Work muſt be to find what the Sum propoſed to be laid out in the Reverſion, would amount to in that Time wherein the Annuity is not to be in Poſſeſſion, as if it were forborn at Intereſt during that Time, at any given Rate* per Cent. (by *Rule* 1. *pag.* 135.) *And the ſecond Work will be to find what Annuity*, &c. *that Amount will purchaſe* (as in *Caſe* 2. *pag.* 151.) *and that will be the Anſwer to the Queſtion.*

Theſe two are the moſt general and uſeful Queſtions that relate to purchaſing Annuities, or Leaſes in Reverſion: Not but that, if there be occaſion, either the Time, or the Rate of Intereſt, may be found by a due Application of their reſpective Rules.

I know that Sir *Samuel Moreland*, and ſeveral other Authors, that treat upon the Subject of Intereſt and Annuities, do aſſert, That neither the Arrears, nor the preſent Worths of Annuities can be truly computed at Simple Intereſt, but only at Compound Intereſt: and therefore, ſay they, all Computations of Annuities, or Leaſes, *&c.* at Simple Intereſt are to be laid aſide as uſeleſs.

'Tis true indeed, that in purchaſing of Annuities, or taking of Leaſes, either to be in preſent Poſſeſſion, or in Reverſion; it is uſual to allow the Purchaſer Compound Intereſt for his ready Money, notwithſtanding it be not lawful to lett out Money at Intereſt upon Intereſt, *viz.* at Compound Intereſt.

However, that we may the better judge whether ſuch Computations as are made of Annuities in Arrears at Simple Intereſt, be uſeful or no: Let us here ſee and compare how near they will agree with the ninety-nine Years Annuities, ſettled by the late Acts of Parliament.

Thoſe Annuities were ſold at ſixteen Years Value, or Purchaſe; that is, 1600 *l.* ready Money bought 100 *l. per annum*, to continue 99 Years, and ſo in proportion for a greater or leſſer Sum.

Now it may be reaſonable to ſuppoſe, that the Parliament deſign'd at the ſettling of thoſe Annuities at that Value, to allow the Purchaſer after the Rate of Six *per cent. per annum* Simple Intereſt for his ready Money, according to the former Laws relating to Intereſt: If ſo, the Intereſt of 1600 *l.* would be 96 *l. per annum* for ever, (as by *Caſe* 1. *pag.* 134.) conſequently there will remain but 4 *l. per annum* for 99 Years, in lieu of the Principal or Purchaſe-Money: that is, there will be but $99 \times 4\,l. = 396\,l.$ repaid inſtead of the 1600 *l.* if it be only receiv'd without any further Conſideration.

But if we compute what that 4 *l. per Annum* would amount to, if it were forborn in Arrears for 99 Years, allowing 6 *per Cent.* for every Payment as it becomes due, *&c.* (*per Caſe* 1. *pag.* 144.) then will that Amount undoubtedly ſhew what Sum will be truly repaid for the 1600 *l.* firſt paid for that Annuity.

And in order to that, there is given $U = 4$, $T = 99$, and $R = 0{,}06$ the Ratio of the Rate of 6 *per Cent.* to find A, the Amount.

Which prepared for the Work is $99 - 1 = 98$
and $\frac{1}{2} R = 0{,}03$

Then $99 \times 98 = 9702$, and $9702 \times 0{,}03 = 291{,}06$; to find A, the Amount.

Next $291{,}06 + 99 = 390{,}06$ and $390{,}06 \times 4 = 1560{,}24$ equal A, *viz.* 1560 *l.* 4 *s.* 9½ *d.* is the Sum that will in effect be received for the 1600 *l.* over and above the 96 *l. per Annum* Intereſt.

Now in this Calculation I take for granted, That if any Yearly Rent be forborn, or let run in Arrears, and any Rate of Simple Intereſt be allow'd for every Payment as it becomes due; the Amount, or Sum of all thoſe Payments will be the very ſame, as if every Year's Rent were actually receiv'd, and immediately put out to Intereſt at the ſame Rate.

Whence it follows, That altho' 396 *l.* be all the Money that's actually received in the ſpace of 99 Years, over or above the 96 *l. per Annum* Intereſt; yet according to the Nature of Intereſt, there is effectually 1560 *l.* 4 *s.* $9\frac{1}{2}$ *d.* receiv'd; ſuppoſing the Annuity to be paid but yearly, which is indeed paid quarterly; and therefore the Sum will be ſtill nearer to 1600 *l.* which plainly ſhews that the Computations of Annuities, or Rents run in Arrears at Simple Intereſt, are not to be wholly rejected as uſeleſs.

But I cannot pretend to ſay much in the behalf of ſuch Computations as are made about the preſent Worth of Annuities, or Leaſes, *&c.* at Simple Intereſt, nor are they to be relied on in Practice: However, becauſe this Tract ſhould not be wanting in that Part, I thought it convenient to ſhew (*altho' but briefly*) how thoſe Computations may be perform'd.

And I ſhall, in the next Chapter, handle the Buſineſs of Purchaſing Annuities, *&c.* more fully; eſpecially that Part of it which relates to Annuities, or Leaſes in Reverſion.

CHAP. III.

The Calculation of Queſtions in Compound Intereſt, *and* Annuities; *perform'd by the help of Logarithms.*

COmpound *Intereſt* is that which is produced from any Principal and its Intereſt put together, as the Intereſt of that Principal becomes due: that is, at every Payment, or rather at the Times when the Payments become due, there is ſtill created a new Principal, by the Increaſe of the growing Intereſt; and therefore it is called *Intereſt upon Intereſt*, or *Compound Intereſt*: which is grounded upon a Series of Geometrical Proportionals continued.

Now in all Computations of this kind, whether they are about Money forborn at Intereſt, or thoſe relating to Annuities, *&c.* we generally make uſe of the Amount or Produce of one Pound and its Intereſt for one Year, inſtead of the Ratio of the given Rate of Intereſt; as before in Simple Intereſt.

And that Amount of 1 *l.* is no more but the Ratio of the given Rate added to a Unit or 1.

For the Ratio is the Intereſt of 1 *l.* (vid. *pag.* 132.) conſequently if that 1 *l.* be added to it, the Sum will be the Amount of 1 *l.* (per *Rule at Caſe* 1. *pag.* 134.) Or

The Amount of 1 *l.* for one Year at any given Rate of Intereſt *per Cent.* may be found by this Proportion;

Viz.

As 100 : 106 :: 1 : 1,06 the *Amount* of 1 *l.* at 6 *per Cent.*
Or 100 : 107 :: 1 : 1,07 the *Amount* of 1 *l.* at 7 *per Cent.*
And ſo on for any other given Rate of Intereſt.

Or the aforeſaid *Amounts* of 1 *l.* may be otherways found by Logarithms,

Thus, { *Add the propoſed Rate of Intereſt to* 100, *and from the Logarithm of that Sum ſubſtract the Logarithm of* 100 (viz. 2.000000) *the Remainder will be the Logarithm of the Amount of* 1 l. *at that Rate of Intereſt* per Cent. }

Example.

Suppoſe the given Rate of Intereſt to be that of 6 per Cent. per Annum..

Then 100 + 6 = 106 its Log. is 2.025306 } Subſtr.
And the Log. of 100 is 2.000000 }
There remains the Logarithm 0.025306 of 1,06

Viz. 1,06 is the Amount of 1 *l.* at 6 *per Cent.* &c.
And ſo for any other Rate of Intereſt.

And becauſe 'tis the Logarithms of thoſe Amounts of 1 *l.* that are of greateſt Uſe in the following Calculations, I have here annexed a ſmall Table of ſeveral Rates of Intereſt, with the Amounts of 1 *l. per Annum* at thoſe Rates, and the Logarithms of thoſe Amounts.

Rates of Intereſt per Cent. l.	*The Amounts of* 1 l. Viz. R.	*Logarithms of thoſe Amounts*, Viz. *of* R.	*Rates of Intereſt per Cent.* l.	*The Amounts of* 1 l. Viz. R.	*Logarithms of thoſe Amounts*, Viz. *of* R.
3	1,03	0.012837	7	1,07	0.029384
4	1,04	0.017033	7½	1,075	0.031408
4½	1,045	0.019116	8	1,08	0.033424
5	1,05	0.021189	8½	1,085	0.035430
5½	1,055	0.023252	9	1,09	0.037426
6	1,06	0.025306	9½	1,095	0.039414
6½	1,065	0.027350	10	1,1	0.041393

Theſe things being underſtood, you may proceed to the Reſolving of Queſtions about Money forborn at Compound Intereſt.

Sect. 1. *Of Compound Intereſt, or Money let out at Intereſt upon Intereſt.*

I Shall here make uſe of the ſame Letters to denote the ſeveral Parts of the Queſtion, as before, except *R* only.

Viz. { *P* denotes any Principal put to Intereſt.
t the Time of its Continuance at Intereſt.
A the Amount or Principal and its Intereſt, }
And *R* denotes the Amount of 1 *l.* for one Year.

If $R =$ the Amount of 1 *l.*, &c. as above;
Then $RR =$ the Amount of 1 *l.* for 2 Years.
$RRR =$ the Amount of 1 *l.* for 3 Years.
$R^4 =$ the Amount of 1 *l.* for 4 Years.
Here $t = 4$, the Index of $RRRR$.
For $1 : R :: R : RR :: RR : R^3 :: R^3 : R^4$ &c. ÷

That is, { As one Pound : Is to the Amount of 1 *l.* at the Year's End :: So is that Amount : To its Amount at two Years, &c. }

Whence it's evident, that the Buſineſs of Compound Intereſt is grounded upon a Series of Terms in Geometrical Proportion, increaſing until the laſt or higheſt Term becomes R^t. And

Then, As 1 *l.* : is to its Amount for any given Time : : So is any propoſed Principal (or Sum) : To its Amount for the ſame Time.

That is, 1 *l.* : R^t : : P : $PR^t = A$, the Amount of P, for the ſame time that $R^t =$ the Amount of 1 *l.* Now from hence ariſes this general

Theorem $PR^t = A$.

This Equation admits of the ſame four Caſes, or Variety of Queſtions, as in that of Simple Intereſt. And here it may not be amiſs to make uſe of the ſame Examples, that we may the more readily ſee the Difference that is between Simple Intereſt and Compound Intereſt.

Caſe 1. If P, t, and R, are given; to find A the Amount.

Rule. *Firſt, Multiply the Log. of R, the Amount of* 1 l. *at the given Rate* per Cent. *with the Time, then to that Product add the Log. of the propoſed Principal, and their Sum will be the Log. of the Amount required.*

Example.

What Sum will 567 l. 10 s. *amount to in nine Years, at the Rate of* 6 per Cent. per Annum?

Hence $R = 1,06$ its Log. 0,025306 } Multiply
And $t =$ 9 }

Their Product is 0,227754 } Add
Alſo $P = 567,5$ its Log. is 2.753966 }

The Sum is the Logarith. 2.981720 of 958,78 $= A$

That is, 958 *l.* 15 *s.* 7 *d.* will be the Amount required. Conſequently 958 *l.* 15 *s.* 7 *d.* — 567 *l.* 10 *s.* = 391 *l.* 5 *s.* 7 *d.* will be the Intereſt only, which is more than the Simple Intereſt of that Principal, for the ſame Time, by 84 *l.* 16 *s.* 7 *d.*

Caſe 2. When A, t, and R, are given; to find P.

Rule. *Multiply the Log. of R, the Amount of* 1 l. *at the given Rate* per Cent. *with the Time* (as before;) *then ſubſtract that Product from the Logar. of the propoſed Amount, and there will remain the Logar. of the Principal required.*

Example.

What Principal or Sum of Money, being put out to Intereſt for nine Years, will amount to (or raiſe the Sum of) 958 l. 15 s. 7 d. *at the Rate of 6* per Cent. per Annum?

Or thus: *There is a Debt of* 958 l. 15 s. 7 d. *which is not due until nine Years hence, but 'tis agreed to be paid in preſent Money; what Sum muſt the Creditor receive, allowing the Rebate or Diſcompt of 6* per Cent. per Annum *to the Debtor for his ready Money.*

Here $A = 958{,}78$ its Log. 2.981720

And $R = 1{,}06$ its Log. is 0.025306 } Multiply

Alſo $t =$ 9

Subſtr. this Log. from the firſt 0.227754

There remains the Logar. 2.753966 of $567{,}5 = P$.

That is, 567 *l.* 10 *s.* is the Principal (or ready Money) as was required.

Caſe 3. Suppoſe P, t, and A, are given; to find R.

Rule. *From the Log. of the propoſed Amount, ſubſtract the Log. of the propoſed Principal; then divide the Remainder by the Time, and the Quotient will be the Logarithm of R, the Amount of* 1 l. &c. *as in this Example.*

Example.

At what Rate of Intereſt per Cent. &c. *will* 567 l. 10 s. *raiſe a Stock, or amount to* 958 l. 15 s. 7 d. *in nine Years Time?*

Here is given $A = 958{,}78$ its Log. 2.981720 } Subſtract

And $P = 567{,}5$ its Logarith. is 2.753966

Alſo $t = 9$ Then 9)0.227754(0,025306 the Log. of $1{,}06 = R$; then $1{,}06 - 1 = 0{,}06$ the Ratio, And it will be, As $1 : 0{,}06 :: $ So is $100 :$ to 6 the Rate of Intereſt *per Cent. &c.*

Caſe 4. Having P, R, and A, given; to find t.

Rule. *From the Log. of the propoſed Amount, ſubſtract the Log. of the propoſed Principal* (as before) *and divide their Difference by the Log. of R, the Amount of* 1 l. *at the given Rate, and the Quotient will ſhew the Time required.*

Example.

In what Time will 567 l. 10 s. *amount to,* (*or raiſe a Stock of*) 958 l. 15 s. 7 d. *at the Rate of* 6 per Cent. per Annum?

Here is given *A* = 958,78 its Log. 2.981720 } Subſtr.
And *P* = 567,5 its Logarithm is 2.753966 }
Alſo *R* = 1,06 its Log. [is 0,025306) 0.227754 (9 = *t*
Viz. Nine Years will be the Time required by the Queſtion.

Theſe four Examples are ſufficient to ſhew how all Queſtions of the like kind may be truly reſolved, if the Time given (*or ſought*) be whole Years; but if it be not, then the odd Time, whether it be under, or above a compleat Year, muſt be reduced into Decimal Parts of a Year, by help of the two Tables in *pag.* 139 and 140, *&c.* and then it will be as eaſy to reſolve any of the foregoing Caſes, when the Time given (or ſought) is in Parts of a Year, as it is for thoſe in whole Years.

Example 1. *When the Time is leſs than a Year, let it be required to find the Amount of* 375 l. *for* 210 *Days, at* 6 per Cent. &c.

In this Queſtion there is given. *P* = 375; *R* = 1,06; and *t* = 210 Days = 0,5753 Decimal Parts of a Year, *per* Table, *pag.* 140. thence to find *A*; as in *Caſe* 1. of this *Section.*

Firſt *R* = 1,06 its Log. is 0.025306 } Multiply
And *t* = 0.5753 }
Their Product is 0.014558 } Add
Again *P* = 375 its Log. is 2.574031 }
The Sum is the Logar. 2.588589 of 387,78 = *A*.

That is, 387 *l.* 15 *s.* 7 *d.* will be the Amount required.

Example 2. *Suppoſe it were required to find what* 265 l. *would amount to in five Years and* 135 *Days, at the Rate of* 6 per Cent. &c.

In this Queſtion is given *P* = 265; *R* = 1,06; and *t* = 5,3698; to find *A* the Amount, as before.

Thus $R = 1{,}06$ its Log. 0.025306 } Multiply
And $t =$ 5.3698 }
Their Product is 0.135888 } Add
Again $P = 265$ its Log. 2.423246 }
The Sum is the Logar. 2.559134 of 362,355 $= A$.
Viz. 362 *l.* 7 *s.* 1 *d.* will be the Amount or Sum required.

Example 3. *Let it be required to find what Principal, or Sum put to Intereſt at* 6 per Cent. *will amount to (or raiſe a Stock of)* 362 l. 7 s. 1 d. *in five Years and* 135 *Days.*

Here is given $A = 362{,}355$; $R = 1{,}06$; and $t = 5{,}3698$; to find P, as in *Caſe* 2.

Thus $A = 362{,}355$ its Log. 2.559134 of the Divid.
And $R = 1{,}06$ its Log. is 0.025306 } Multiply
$t =$ 5.3698 }
Then ſubſtract this Logar. 0.135888 from the firſt Log.
And there remains the Log. 2.423246 of $265 = P$.
Viz. 265 *l.* will be the Principal, or Sum required.

If the Work of theſe three Examples be well underſtood, it muſt needs be eaſy to find proper Anſwers to all Queſtions that can be propoſed in the other Caſes, *viz.* for any propoſed Time, and Rate of Intereſt, according as the Date requires.

Sect. 2. *Of Annuities, or Rents,* &c. *that are in Arrears, computed at Compound Intereſt.*

I Shall here make uſe of the ſame Letter to repreſent the ſame things as in *Pag.* 143. ſave only that R doth here denote the Amount of 1 *l.* as in the laſt Section.

Viz. { $U =$ the Annuity, or Rent.
$t =$ the Time of its being unpaid.
$A =$ the Amount, or Sum of all the Arrears.

And $R =$ the Amount of 1 *l.* *&c.* as in Simple Intereſt.

Then will $A - U = AR - UR^t$.

From this Equation is deduced the four following Caſes, and the Rules by which they are reſolved.

N.B. *The following Caſes admit of ſome Variety, in reſpect to the true Intervals of the Times, which are betwixt the ſeveral Payments*; viz. *it muſt be well conſidered whether the Payments are to be made Yearly, or at the End of ſome part of a Year; as the* $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{12}$, $\frac{1}{52}$, $\frac{1}{365}$. *That is*, Half-yearly, Quarterly, Monthly, &c. *the which muſt be very carefully minded when the Queſtion is propoſed, in any of the following Caſes, or the Anſwers will not be true.*

Caſe 1. Having U, t, and R, given; to find A.

That is, If any Annuity, or Rent, *&c.* with the Time of its being unpaid, and the Rate of Intereſt *per Cent.* are given; thence to find what Sum all thoſe Arrears will amount to in that Time, allowing Compound Intereſt for every particular Payment, as it becomes due.

Rule. *Multiply the Log. of R (the Amount of* 1 l.) *with the Time, and to that Product add the Log. of the Annuity; then find the Number which belongs to that Sum, and ſubſtract the Annuity from that Number:* (So far the Work is general to all Times of Payment.) *Then, if the Remainder* (*) *be divided by R* — 1 (viz. *by the Ratio of the Rate of Intereſt* per Cent.) *the Quotient will ſhew the Sum, or Amount of all the Arrears for Yearly Payments.*

Example 1.

Suppoſe 356 l. *Annuity, or Yearly Rent, were forborn or unpaid nine Years; what would the Sum of all thoſe Arrears amount to at 6* per Cent. &c?

Here is given $U = 356$; $t = 9$; and $R = 1{,}06$; to find A.

$$\left.\begin{array}{r} \text{Thus } R = 1{,}06 \text{ its Log. is } 0.025306 \\ t = \underline{\qquad\quad 9} \end{array}\right\}\text{Multiply}$$

$$\left.\begin{array}{r} \text{The Product is } 0.227754 \\ \text{Again } U = 356 \text{ its Log. is } \underline{2.551450} \end{array}\right\}\text{Add}$$

The Sum is the Logarithm 2.779204 of 601,4545

And $601{,}4545 - 356 = 245{,}4545$; $R - 1 = 0{,}06$

Then 0,06) 245,4545 (4090,91 $= A =$ 4090 *l.* 18 *s.* 2 $\frac{1}{2}$ *d.* being the Sum of all the Arrears for Yearly Payments.

Note, *Here we may obſerve the Difference that is betwixt the Arrears computed at Simple Intereſt, and at Compound Intereſt.*

Or, the Amount may be alſo found by Logarithms without the laſt Diviſion; for having once form'd the Dividend, and Diviſor, (as before) then the Difference of their Logarithm will ſhew the Amount.

Thus 245,4545 its Log. 2.389972 } Subſtract
And $R - 1 = 0{,}06$ its Log. 8.778151 }
There remains the Log. 3.611821 of $4090{,}91 = A$.

And ſo on, as before.

But if the Payments are to be made at any Time leſs than a compleat Year, as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{12}$, *&c.*

Then, { *You muſt take the like part of the aforeſaid Remainder* (*) *and make it a new Dividend: And you muſt alſo take the ſame part of the Logarithm of R; and the Number anſwering to that part being made leſs by an Unit or* 1, *will be a new Diviſor (inſtead of R* − 1) *by which the new Dividend being divided, the Quotient will ſhew the Sum of all the Arrears required.*

Example 2. In Half-yearly Payments.

Suppoſe the aforeſaid 356 l. per Annum *were to be paid by Half-yearly Payments,* viz. 178 l. *every Half-year, and it were unpaid nine Years* (as before) *what Sum would all thoſe Arrears amount to then, at* 6 per Cent. &c.?

Here (as bef.) $R = 1{,}06$ its Log. 0.025306 } Multiply
And $t =$ 9 }
Their Product is 0.227754 } Add
Alſo $U = 356$ its Logarithm is 2.551450 }
The Sum is the Logarithm 2.779204 of 601,4545

and $601{,}4545 - 356 = 245{,}4545$. So far the Work is juſt the ſame as before for whole Years:

But here the $\frac{1}{2}$ of $245{,}4545 = 122{,}72725$ is the new Dividend:

Next, $R = 1{,}06$ the $\frac{1}{2}$ of its Log. is 0.012653 whoſe Number is 1,029563; and $1{,}029563 - 1 = 0{,}029563$ for the New Diviſor.

Then 0,029563) 122,72725 (4151,38 = *A.*

That is, 4151 *l.* 7 *s.* 7 *d.* is now the Amount or Sum of all the Arrears; which is 60 *l.* 9 *s.* 4¼ *d.* more than the Amount in the laſt Example for whole yearly Payments.

Here alſo the Amount may be obtain'd by Logarithms without the laſt Diviſion; having once form'd the New Dividend, and its proper Diviſor as before.

Thus, 122,727 its Log. is 2.088938 } Subſtract.
And 0,029563 its Log. is $\overline{8}$.470748 }
There remains the Log. 3.618190 of 415138 = *A.*
Viz. 4151 *l.* 7 *s.* 7 *d.* = *A*, the Amount, &c. as before.

Example 3. In Quarterly Payments.

Let it be required to find the Amount or Sum of all the Arrears that would be due from 245 l. per Annum, *to be paid by Quarterly Payments* (viz. 61 l. 5 s. per Quarter) *ſuppoſing it to be forborn, or unpaid ten Years and a half, at* 5½ per Cent. &c.

Here is given *R* = 1,055 its Log. 0.023252 } Multiply
And *t* = 10,5 }
The Product is 0.244146 } Add
Again *U* = 245 its Log. is 2.389166 }
Their Sum is the Logarith. 2.633312 of 429,846

And 429,846 — 245 = 184,846 So far the Work is ſtill the ſame as in the whole Years.

But now it muſt be the ¼ of 184,846 = 46,2115 for the New Dividend.

Next, *R* = 1,055 the ¼ of its Log. is 0.005813 whoſe Number is 1.01348; and 1.01348 — 1 = 0.01348 for the new Diviſor.

Then 0.01348) 46,2115 (3428,15 = *A.*

That is, 3428 *l.* 3 *s.* will be the Amount or Sum of all thoſe Arrears, as was required.

Or the laſt Diviſion may be avoided, and the Amount may be found by the Logarithms: Thus,

Dividend 46,2115 its Log. 1.664750 } Subſtract
Diviſor is 0.01348 its Log. $\overline{8}$.129690 }
There remains its Logar. 3.535060 of 3428.15 = *A.*
Viz. 3428 *l.* 3 *s.* is the Amount required, as above.

The ſame may be done for Monthly, Weekly, or Daily Payments, Care being taken in forming the new Dividend, and Diviſor, according to their reſpective Time.

Caſe 2. When A, t, and R, are given; to find U.

That is, To find what Annuity, or yearly Rent, being forborn or unpaid, any aſſigned Time, will amount to a propoſed Sum, allowing any given Rate of Intereſt *per Cent.* for every Payment as it becomes due.

Note, *This Caſe, as well as the laſt, admits of ſome Variety in reſpect to the Times of Payment*; viz. *Whether they are* Yearly, Half-yearly, *or* Quarterly, &c. *the which muſt be carefully obſerved, as the following Rule directs.*

Rule. *Multiply the Log. of R* (the Amount of 1 *l.*) *with the given Time, and the Number anſwering to their Product being made leſs by* 1, *will be the Diviſor*, (for all Times of Payment at that Rate of Intereſt.)
Next, Multiply the propoſed Amount or Sum, with R — 1, viz. *with the Ratio of the given Rate: Then divide their Product by the aforeſaid Diviſor, and the Quotient will ſhew the Annuity or Rent for Yearly Payments.*

Example 1.

What Annuity, or Yearly Rent, being forborn or unpaid nine Years, will amount to (or raiſe a Stock of) 4090 l. 18 s. 2 ½ d. *at the Rate of 6* per Cent. *for every Payment as it becomes due.*

Here is given $A=4090{,}91$; $t=9$; and $R=1{,}06$; to find U.

$$\left.\begin{array}{r} \text{Firſt } R=1{,}06 \text{ its Log. is } 0.025366 \\ \text{And } t= \quad 9 \end{array}\right\}\text{Multiply}$$

The Product is the Logar. 0.227754 of 1,68948
And $1{,}68948-1=0{,}68948$ for the Diviſor.

$$\left.\begin{array}{r} \text{Again } A=4090{,}91 \\ \text{And } R-1= \quad 0{,}06 \end{array}\right\}\text{Multiply}$$

The Product is 245,4546 for the Dividend.

Then 0,68948) 245,4546 (356 $=U$.

That is, 356 *l.* will be the Annuity, or Yearly Rent, as was required to be paid by Yearly Payments.

Or having found the Diviſor (*as above*) the reſt of the Work may be perform'd by Logarithms only,

Thus, $A = 4090,91$ its Log. 3.611821 } Add
And $R - 1 = 0,06$ its Log. 8.778151 }
Sum is 2.389972 } Subſtract
The Diviſ. is 0,68948 its Log. 9.838522 }
There remains the Logarith. 2.551450 of $356 = U$.
Viz. 356 *l.* will be the Yearly Annuity, or Rent, &c.

But if the Payments are required to be paid either Half-yearly, or Quarterly, &c. as in the laſt Caſe:

Then { *Inſtead of $R - 1$, you muſt take the reſpective Part of the Logarithm of R* (viz. of the Amount of 1 *l.*) *and with the Number anſwering to that Part, being made leſs by* 1, *multiply the propoſed Amount; then divide that Product by the aforeſaid Diviſor, and the Quotient will ſhew what Annuity or Rent muſt be paid at the propoſed Time of each Payment.*

Example 2. In Half-yearly Payments.

What Annuity, or Rent, to be paid by Half-yearly Payments, being forborn or unpaid nine Years, will amount to the Sum of 4151 l. 7 s. 7 d. *allowing* 6 per Cent. *for every Payment as it becomes due?*

Here is given $A = 4151,38$; $t = 9$; and $R = 1,06$; to find U.

Firſt $R = 1,06$ its Log. is 0.025306 } Multiply
And $t =$ 9 }
The Product is the Log. 0.227754 of 1,68948
And $1,68948 - 1 = 0,68948$ for the Diviſor, as before in the laſt Example.

But inſtead of the Multiplicator $R - 1$, it muſt be $R = 1,06$ the $\frac{1}{2}$ of its Log. 0,012653 its Numb. is 1,029563 and $1,029563 - 1 = 0,029563$ the Multiplicator.

And it muſt be $A = 4151,38$
Multiplied with 0,029563
The Product is 122,72724 the new Dividend.

Then 0,68948) 122,72724 ($178 = U$, viz. 178 *l.* will be the Half-yearly Payments of the Annuity, or Rent required.

Or having found the Diviſor, and the new Multiplicator uſed inſtead of $R-1$; then the Work may be done by the Logarithms,

Thus, $A = 4151,38$ its Log. 3.618190 } Add

Multipl. is 0,029563 its Log. 8.470749

Sum 2.088939 } Subſtract

Diviſor is 0,68948 its Log. is 9.838520

There remains the Logarith. 2.250419 of $178 = U$

Viz. 178 *l.* is the Half-yearly Payments, &c. as before.

Example 3. In Quarterly Payments.

Let it be required to find what Annuity, or Rent to be paid by Quarterly Payments, being forborn or unpaid ten Years and a half, would amount to the Sum of 3428 l. 3 s, *at Five and a half* per Cent. per Annum, *for every Payment as it becomes due?*

Here is given $A = 3428,15$; $t = 10,5$; and $R = 1,055$; to find U, the Quarterly Payments.

Firſt $R = 1,055$ its Log. is 0.023252 } Multiply

And $t =$ 10,5

The Product is the Logar. 0.244146 of 1.75447

Then $1,75447 - 1 = 0,75447$ the Diviſor.

Again $R = 1,055$ the $\frac{1}{4}$ of its Log. is 0.005813 its Number is 1,01348; and $1,01348 - 1 = 0,01348$ the Multiplicator.

Next $A = 3428,15$

Multiplied with 0,01348

Produces 46,21146 the Dividend.

Then 0,75447) 46,21146 ($61,25 = U$.

That is, 61 *l.* 5 *s.* will be the Quarterly Rents or Payments; conſequently $61,25 \times 4 = 245$ *l. per Annum* will be the Annuity required.

Or by Logarithms, thus

$A = 3428,15$ its Log. is 3.535060 } Add

And 0,01348 its Log. is 8.129690

Sum is 1.664750 } Subſtract

Again 0,75447 its Log. 9.877642

There remains the Log. 1.787108 of $61,25 = U$

Viz. 61 *l.* 5 *s.* will be the Annuity, or Rent to be paid every Quarter, &c.

The like may be done in finding the Monthy, Weekly or Daily Annuities, care being taken in forming the new Multiplicators, and Diviſors proper for the Time given.

Caſe 3. Having *U*, *A*, and *R* given; to find *t*.

That is, to find the Time in which any propoſed Annuity, or Rent, will amount to any aſſign'd Sum, at any given Rate of Intereſt *per Cent.* &c.

Rule. { *Multiply the propoſed Amount with R* — 1, viz. *with the Ratio of the given Rate; and divide that Product by the Annuity, and to the Quotient add* 1. *Then if the Logarithm of that Sum be divided by the Logarithm of R* (*viz.* the Amount of 1 *l.*) *the Quotient will ſhew the Time for Yearly Payments.*

Example 1.

In what time will 356 l. *Yearly Rent, or Annuity, amount to the Sum of* 4090 l. 18 s. 2 ½ d. *at the Rate of* 6 per Cent. &c.?

Here is given $U = 356$; $A = 4090,91$; and $R = 1,06$; to find *t*.

Thus, $A = 4090,91$ }
And $R - 1 = 0,06$ } Multiply

$U = 356$) 245,4546 (0,68948

And $0,68948 + 1 = 1,68948$ its Log. is 0,227754

Then $R = 1,06$ its Log. 0,025306) 0,227754 ($9 = t$.

Viz. 9 Years will be the Time required in the Queſtion.

Or otherwiſe by Logarithms,

Thus $A = 4090,91$ its Log. is 3.611821 }
And $R - 1 = 0,06$ its Log. $\bar{8}$.778151 } Add

Sum 2.389972 }
Again, $U = 356$ its Log. is 2.551450 } Subſtract

There remains the Logar. $\bar{9}$.838522 of 0,68948

And $0,68948 + 1 = 1,68948$ its Log. 0,227754. Then, as before, 1,06 its Log. 0,025306) 0,227754 ($9 = t$.

Viz. 9 Years will be the Time required, if the Payments are to be paid but once a Year.

But if they are to be paid either *Half-yearly* or *Quarterly*, &c. as before in the two laſt Caſes;

Then, *Inſtead of making* $R-1$ *the Multiplicator as above, you muſt take ſuch a part of the Logarithm of R the Amount of* 1 l. *as the given Time requires; and with its Number made leſs by* 1, *multiply the propoſed Amount: Then divide that Product by the Sum which ought to be paid at its reſpective Time, and to that Quotient add* 1; *and ſo proceed as the Rule directs.*

Example 2. In Half-yearly Payments.

In what Time will 356 l. per Annum *amount to the Sum of* 4151 l. 7 s. 7 d. *if it is to be paid by Half-yearly Payments,* viz. 178 l. *every Half-year; and to be allow'd* 6 per Cent. &c. *for every Payment as it becomes due?*

Here is given $U=178$, *viz.* the ½ of 356, $A=4151,38$; and $R=1,06$; to find t, the Number of Years.

Firſt $R=1,06$ the ½ of its Log. is 0.012653, and its Number is 1,029563; then 1,029563 — 1 = 0,029563 the Multiplicator inſtead of $R-1$.

And it will be $A=4151,38$
Multiplied with 0,029563
The Product is 122,72724 for the Dividend.

Then $U=178$) 122,72724 (0,68948; to which
Add 1, and it will be 1,68948 its Log. is 0.227754
And $R=1,06$ its Log. is 0.025306) 0,227754 ($9=t$
Viz. 9 Years will be the Time ſought.

Or having once form'd the Multiplicator, then the Work may be done by Logarithms,

Thus $A=4151,38$ its Log. is 3.618190 } Add
Multiplic. is 0,029563 its Log. 8.470749 }
Sum 2.088939 } Subſtract
Again $U=178$ its Log. is 2.250418 }
There remains the Logarith. 9.838521 of 0,68948.

And 0,68948 + 1 = 1,68948 its Log. is 0.227754; $R=1,06$; its Log. is 0.025306) 0,227754 ($9=t$, the Number of Years ſought, as before.

Example 3. In Quarterly Payments.

In what Time will 245 l. per Annum, *to be paid Quarterly* (viz. 61 l. 5 s. *per Quarter*) *amount to the Sum of*

3428 l. 3 s. *allowing* $5\frac{1}{2}$ per Cent. *for the Forbearance of the Payments as they become due?*

Here is given $U = 61,25$ (*viz.* $\frac{1}{4}$ of 245 *l.*) $R = 1,055$;
and $A = 3428,15$; to find t, the Time.

Firſt $R = 1,055$; the $\frac{1}{4}$ of its Log. is 0.005813; its Number is 1,01348; and $1,01348 - 1 = 0,01348$ the *Multiplicator*; and $A = 3428,15$ the *Multiplicand*,
Multiplied with 0.01348
Their *Product* is 46,21146 the *Dividend*.

Then $U = 178)\,46,21146\,(0,75447$ to which
Add 1, and it is 1,75447 its Log. is 0.244146:

Then $R = 1,055$ its Log. $0.023252)\,0.244146\,(10,5 = t$
Viz. Ten Years and a half will be the Time required by the Queſtion.

The ſame may be found by Logarithms, as in the laſt Example.

Caſe 4. Having U, t, and A, given; to find R.

That is, If any Annuity, with the Time of its being unpaid, and the Sum it's propoſed to amount to in that Time, are given; thence to find the Rate of Intereſt *per Cent.* that muſt be allow'd for every Payment as it becomes due.

The Solution of this Queſtion is more difficult than any hitherto has been; and the Equation by which the Value of R muſt be found, is this, $\frac{A}{U} - 1 = \frac{A}{U}R - R^t$, which is called an Adfected Equation, and requires an Analytical Solution to find the Value of R, which cannot otherwiſe be found by any general *Rule*, that can be preſcribed here, as in the precedent Caſes, but only by approaching to it with Trials. And in order to perform that Approximation with the leaſt trouble, I ſhall here propoſe the following Method, as the eaſieſt I can at preſent think of.

Which is thus:

Firſt, Divide the propoſed Amount, by the Annuity and reſerve the Quotient. Next, Subſtract 1 *from that Quotient, and call the Remainder* x.

Then you muſt make choice (by gueſs) *of ſuch a Number for the Value of* R, *as you think will be the Amount of*

1 l. *at the Rate of the Intereſt ſought; and multiply that Number into the reſerved Quotient: Alſo you muſt multiply the Logarithm of that ſuppoſed Number, with the Time; and find the Number which belongs to their Product. Then, if the Difference between that Number and the aforeſaid Product of R into the reſerved Quotient, be equal to* (*viz.* be the ſame with) *the Number call'd* x, *you have hit upon the true Value of R. But if they are not equal, then another Trial muſt be made,* &c. *as in the following Examples.*

Example 1. In Yearly Payments.

There is an Annuity, or Rent of 356 l. per Annum, *and it has been forborn or unpaid nine Years; and there is* 4090 l. 18 s. $2\frac{1}{2}$ d. *given for the Amount, or Sum of all the Arrears: it's required to find what Rate of Intereſt* per Cent. *is allow'd for each Payment as it became due?*

Here is given $U=356$; $t=9$; and $A=4090,91$; to find R, the Amount of 1 *l.*

Thus 356) 4090,91 (11,49132 the Quotient to be reſerv'd; from which ſubſtract 1, and it will be 10,49132; which is the Number call'd x: And ſo far the Work will always be certain, it being only that of the firſt Side of the Equation.

Next, I will ſuppoſe the Value of R to be 1,05; and then the reſerved Quotient 11,49132 × 1,05 the Product will be 12,065886.

$$\begin{array}{lr} \text{Again, If } R=1{,}05 \text{ its Log.} & 0.021189 \\ \text{And } t= & 9 \\ \hline \end{array}\Bigg\}\text{Multiply}$$

The Product is the Logar. 0.190701 of 1,55132

Then $12,065886 - 1,55132 = 10,51456$; which is more than 10,49132 the Number call'd x: Therefore this ſuppoſed Value of $R = 1,05$ is taken too little.

Again, For a ſecond Trial I will ſuppoſe $R=1,07$; and then the reſerved Quotient 11,49132 × 1,07 the Product will be 12,29571.

$$\begin{array}{lr} \text{And if } R=1{,}07 \text{ its Log. is} & 0.029384 \\ \text{And } t= & 9 \\ \hline \end{array}\Bigg\}\text{Multiply}$$

The Product is the Logar. 0.264456 of 1,83847

Then $12,29571 - 1,83847 = 10,45724$; which is leſs than 10,49132 the Number call'd x; And therefore I conclude the laſt $R = 1,07$ was taken too big, and that the Value of R is ſome Number between 1,05 and 1,07.

Let us therefore take $R = 1,06$ (viz. *in the middle between* 1,05 *and* 1,07.) And then the reſerved Quotient 11,49132 × 1,06 the Product will be 12,180799.

And if $R = 1,06$ its Log. is 0.025306 } Multiply
And $t = 9$ }
The Product is the Logar. 0.227754 of 1,689479

Then 12,180799 − 1,689479 = 10,49132 ; which is juſt the ſame with the Number call'd x.

Whence I conclude that the true Value of $R = 1,06$ and $1,06 - 1 = 0,06$ is the Ratio of the Rate of Intereſt ſought.

Then it will be, As 1 : is to 0,06 : : So is 100 : to 6, the Rate of Intereſt *per Cent.* as was required for Yearly Payments.

But if the Payments are to be made either Half-yearly or Quarterly, &c.

Then { *Inſtead of multiplying the reſerved Quotient, with the ſuppoſed Value of* R (as above) *you muſt take ſuch a part of the Log. of that* R, (*viz.* of the aſſumed Amount of 1 *l.*) *as the given Time requires, and with its Number multiply the aforeſaid reſerv'd Quotient, and then proceed on as before.*

Example 2. In Half-yearly Payments.

Suppoſe the ſame Annuity of 356 l. per Annum *were to be paid Half-yearly* (*viz.* 178 *l.* every Half-year) *if it be forborn nine Years, and then amounts to the Sum of* 4151 l. 7 s. 7 d. *What Rate of Intereſt* per Cent. *is allow'd*, &c.?

Here is given $U = 178$ (viz. *the* ½ *of* 356 l.) $t = 9$, and $A = 4151,38$; to find R, the Amount of 1 *l.* &c.

Firſt 178) 4151,38 (23,32236 the reſerv'd Quotient, and $23,32236 - 1 = 22,32236$ the Number call'd x.

So far is the ſame as before in whole Years.

Now here I will aſſume the firſt Value of $R = 1,06$. Then if $R = 1,06$ the ½ of its Log. 0.012653 ; and the Number to that Half Log. is 1,029563 the Multiplicator inſtead of $R = 1,06$ as before : And then the reſerved Quotient 23,32236 × 1,0295663, the Product will be 24,011839.

Again, If $R = 1{,}06$ its Log. is $\left.\begin{array}{r}0.025306\\ \text{And } t = \underline{\qquad 9}\end{array}\right\}$ Multiply

Their Product is the Logar. 0.227754 of 1,689479

Then $24{,}011839 - 1{,}689479 = 22{,}32236$; which is juſt the ſame with 22,32236 the Number call'd x. And therefore the true Value of $R = 1{,}06$: And ſo on for the Rate *per Cent.* as in the laſt Example.

Example 3. In Quarterly Payments.

An Annuity of 245 l. per Annum, *to be paid Quarterly* (viz. 61 *l.* 5 *s.* every Quarter) *is forborn ten Years and a half; and then 'tis ſaid to amount to the Sum of* 3428 l. 3 s. *it is required to find what Rate of Intereſt* per Cent. *is allow'd*, &c.

Now here is given $U = 61{,}25$ (viz. *the* ¼ *of* 245 l.) $t = 10{,}5$; and $A = 3428{,}15$; to find R, the Amount of 1 *l.* *&c.*

Firſt 61,25) 3428,15 (55,96979 the reſerved Quotient, and $55{,}96979 - 1 = 54{,}96979$ the Number call'd x.

Then let us here ſuppoſe $R = 1{,}05$ and then the ¼ of its Log. will be 0.005297 whoſe Number is 1,01227 the Multiplicator.

And the reſerved Quotient $55{,}96979 \times 1{,}01227$ the Product will be 56,65754.

Again, If $R = 1{,}05$ its Log. is $\left.\begin{array}{r}0.021189\\ \text{And } t = \underline{\qquad 10{,}5}\end{array}\right\}$ Multiply

The Product is the Logarith. 0.222484 of 1,66911

Then $56{,}65754 - 1{,}66911 = 54{,}98843$ which is a little more than 54,96979 the Number call'd x. Therefore the Value of $R = 1{,}05$ is taken ſomething too little.

And making a ſecond Trial, by ſuppoſing $R = 1{,}06$; and then proceeding on as before, I find the Reſult to be about as much leſs than 54,96979 the Number call'd x, as the laſt found Number was too much.

I ſhall therefore aſſume $R = 1{,}055$; *viz.* in the middle between 1,05 and 1,06.

And then if $R = 1{,}055$ the ¼ of its Log. 0.005813; and its Number is 1,01348 for the Multiplicator.

Then the reſerved Quotient $55{,}96979 \times 1{,}01348$ the Product is 56,72426.

Again, If $R = 1,055$ its Log. is 0.023252 } Multiply
And $t =$ 10,5 }

The Product is the Logarith. 0.244146 of 1,75447

Then 56,72426 — 1,75447 = 54,96979 which is exactly the ſame with the Number call'd x.

And therefore I conclude that the true Value of $R = 1,055$ and $1,055 - 1 = 0,055$ the Ratio of the Rate. And

Then it will be, as 1 : is to 0,055 :: ſo is 100 : to 55. *Viz.* Five and a Half is the Rate of Intereſt *per Cent. per Annum*, as was required.

What hath been here done by Multiplication, and Diviſion, may, if you pleaſe, be perform'd by Logarithms, as in the precedent Caſes.

As for inſtance in the laſt Example;

Wherein $A = 3428,15$ its Log. 3.535060 } Subſtract
And $U = 61,25$ its Log. is 1.787108 }
The Remainder is the Log. 1.747952 of 55,9698
which is here the reſerved Quotient.

And $55,9698 - 1 = 54,9698$ the Number call'd x.

Then if $R = 1,055$ the $\frac{1}{4}$ of its Log. 0.005813 } Add
the Log. of the reſerv'd Quotient 1.747952 }
The Sum is the Logarithm 1.753765 of 56,7243
Again, If $R = 1,055$ its Log. is 0.023252 } Multiply
And $t =$ 10,5 }
The Product is the Logarithm 0.244146 of 1,75447

Then 56,7243 — 1,75447 = 54,96983 is juſt the ſame with the Number call'd x; and therefore, &c. as above.

Sect. 3. *To find the Preſent Worth of Annuities, Penſions, or Leaſes, &c. at Compound Intereſt.*

THE Method of Calculating the preſent Worth of Annuities, &c. is grounded upon a Series of Geometrical Proportionals continually decreaſing; as in the following Series. In order to that,

Let { U denote the *Annuity*, or *Rent*. } As before, in Simple Inter.
{ t the *Time* of its *Continuance*. }
{ R the *Amount* of 1 *l.* }

And P the *Preſent Worth* of the *Annuity*; And is the Sum of all the Terms in the Series, which will run thus:

$$\frac{U}{R} : \frac{U}{RR} :: \frac{U}{RR} : \frac{U}{RRR} :: \frac{U}{RRR} : \frac{U}{R^4}, \text{ \&c. in } ∺$$

until the laſt Term be $\frac{U}{R^t}$.

Then $P - \frac{U}{R^t}$ will be the Sum of all the Antecedents.

And $P - \frac{U}{R}$ will be the Sum of all the Conſequents.

And then it will be $\frac{U}{R} : \frac{U}{RR} :: P - \frac{U}{R^t} . P - \frac{U}{R}$.

See the Compendium of Algebra, Chap. 6.

The Product of the Extremes into the Means, will produce this

Theorem, $PR^t = \frac{UR^t - U}{R - 1}$.

From this Equation are deduced the following Rules, which alſo admit of four Caſes.

Caſe 1. Having U, t, and R given; to find P.

That is, If any propoſed Annuity, or Yearly Rent, and the Time of its Continuance are given; to find the preſent Worth of that Annuity, allowing any Rate of Compound Intereſt *per Cent.* to the Purchaſer, &c.

Rule. *Multiply the Log. of R* (the Amount of 1 *l.*) *with the given Time, and from the Number which belongs to that Product, ſubſtract* 1: *Then multiply the Remainder with the Annuity, and the Product will be a Dividend.* *Then multiply the Number which belong'd to the firſt Product, with* $R - 1$ (*viz.* with the Ratio of the Rate) *and make their Product a Diviſor; the Quotient ariſing from thence will ſhew the preſent Worth for Yearly Payments.*

Example 1. In Yearly Rents.

Suppoſe a Leaſe of 250 l. per Annum *were to be lett for* 21 *Years; what may the preſent Worth of that Leaſe be, at the Rate of* 5 per Cent. &c.?

In this Queſtion there is given $U=250$, $t=21$, and $R=1{,}05$; to find P, the preſent Worth for Yearly Rents.

Firſt $R=1{,}05$ its Log. is 0.021189 } Multiply
And $t=$ 21

The Product is the Logar. 0.444969 of 2,7859
Then $2{,}7859-1=1{,}7859$; and $1{,}7859\times250=446{,}475$ for the Dividend.

Again $R-1=0{,}05$; and $2{,}7859\times0{,}05=0{,}139295$ for the Diviſor.

Then 0,139295) 446,475 ($3205{,}2478=P$.

That is, 3205 *l.* 5 *s.* will be the preſent Worth of ſuch a Leaſe as was required.

Or the ſame may be otherwiſe perform'd by Logarithms thus:

Firſt $R=1{,}05$ its Log. is 0.021189 } Mult. as before.
And $t=$ 21
The Product is 0.444969 } Add
Again $U=250$ its Log. 2.397940

The Sum is the Logarithm 2.842909 of 696,48; from which ſubſtract the Annuity 250, and there will remain 446,48 its Logarithm is 2.649802
The Log. of R mult. with t, is 0.444969 } Add
And $R-1=0{,}05$ its Log. is 8.698970
From the up. Log. take this Log. $\bar{9}$.143939
There remains the Logarith. 3.505863 of 3205,25

That is, 3205 *l.* 5 *s.* will be the preſent Worth required by the Queſtion, as before; ſuppoſing the Rents to be paid but once a Year.

But if the Payments of the Annuity, or Rents, *&c.* are to be made at any Time leſs than a compleat Year, as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{12}$, *&c.* that is, *Half-yearly*, or *Quarterly*, &c. Then the Diviſor muſt be otherwiſe form'd than by $R-1$.

Thus { *You muſt take ſuch a part of the Log.* R *the Amount of* 1 l. *as the given Time requires, and make uſe of the Number anſwering to that Part, made leſs by* 1, *inſtead of* R − 1 *the Ratio, &c. as in the following Examples.*

Exam-

Example 2. For Half-yearly Rents.

Suppoſe the aforeſaid 250 l. per Annum *were to be paid Half-yearly (viz.* 125 l. every Half-year*) and a Leaſe of it were to be lett for* 21 *Years; what would the preſent Worth of that Leaſe be, at the rate of Five* per Cent. &c.

Here is given $U = 125$, $t = 21$, and $R = 1{,}05$; to find P.

$$\left.\begin{array}{lr}\text{Firſt } R = 1{,}05 \text{ its Log. is} & 0.021189 \\ \text{And } t = & \underline{\quad 21}\end{array}\right\}\text{Multiply}$$

The Product is the Logarith. 0.444969 of 2,7859

Then $2{,}7859 - 1 = 1{,}7859$; and $1{,}7859 \times 125 = 223{,}2375$ for a Dividend, Which is only the Half of the Dividend in the laſt Example.

Next $R = 1{,}05$ the $\frac{1}{2}$ of its Log. is 0.010594 whoſe Number is 1,0247; and $1{,}0247 - 1 = 0{,}0247$ to be the Multiplicator inſtead of $0{,}05 = R - 1$.

That is, $2{,}7859 \times 0{,}0247 = 0{,}06881$ will be the Diviſor.

Then $0{,}06881)\,223{,}2375\,(3244{,}25 = P$.

Viz. 3244 *l.* 5 *s.* will be now the preſent Worth of that Leaſe, which is more by 39 *l.* than that in the laſt Example for Yearly Payments.

And by the way, we may here obſerve the great Difference that is betwixt the preſent Worth of any Annuity, or Leaſe, &c. *computed at* Simple Intereſt, *and the ſame at* Compound Intereſt. *See the Example in* pag. 150.

The ſame may alſo be performed by Logarithms as in the laſt Example, having found the Multiplicator inſtead of $R - 1$, as above.

$$\left.\begin{array}{lr}\text{Then } R = 1{,}05 \text{ its Log. is} & 0.021189 \\ \text{And } t = & \underline{\quad 21}\end{array}\right\}\text{Multiply}$$

$$\left.\begin{array}{lr} & 0.444969 \\ \text{Again } U = 125 \text{ its Log. is} & \underline{2.096910}\end{array}\right\}\text{Add}$$

The Sum is the Logarithm 2.541879 of 348,24

From this 348,24 ſubſtract the Rent 125, and there remains 223,24 its Logarithm is $\underline{2.348772}$

$$\left.\begin{array}{lr}\text{The Log. of } R\text{, multip. with } t\text{, is} & 0.444969 \\ \text{The new Multip. } 0{,}0247 \text{ its Log.} & \underline{8.392697}\end{array}\right\}\text{Add}$$

From the up. Log. ſubſt. this Log. $\underline{\bar{8}.837666}$

There remains the Logarithm 3.511106 of 3244,2

Viz. 3244 *l.* 4 *s.* is here found to be the preſent Worth, which is leſs than that found by the Pen by 1 *s.* being but very inconſiderable.

Example 3. In Quarterly Rents.

What is 80 l. per Annum, *to continue* 31 *Years and a half, worth in preſent Money, at the Rate of* 7 per Cent. per Annum; *if the Rents be paid Quarterly*, (*viz.* 20 *l.* every Quarter of a Year?)

In this Queſtion $U = 20$, $t = 31{,}5$, and $R = 1{,}07$; to find P, as before.

Firſt $R = 1{,}07$ its Log. is 0.029384 } Multiply
And $t =$ 31,5 }

The Product is the Logarith. 0.925596 of 8,4255

And $8{,}4255 - 1 = 7{,}4255$ and $7{,}4255 \times 20 = 148{,}51$ for the Dividend.

Again $R = 1{,}07$ the $\frac{1}{4}$ of its Log. is 0.007346 whoſe Number is 1,01706; and $1{,}01706 - 1 = 0{,}01706$.

Next $8{,}4255 \times 0{,}01706 = 0{,}143739$ the Diviſor.

Then 0,143739) 148,510000 (1033,185 $= P$.

That is 1033 *l.* 3 *s.* 8$\frac{1}{2}$ *d.* will be the preſent Worth as was required.

Or having found the Multiplicator 0,01706, as above; then the reſt of the Work may be perform'd by Logarithms,

Thus, $R = 1{,}07$ its Log. is 0.029384 } Multiply
And $t =$ 31,5 }

9.925596 } Add
Again, $U = 20$ its Log. is 1.301030 }
The Sum is the Logarith. 2.226626 of 168,51

From this 168,51 ſubſtract the Quarterly Rent 20, remains 148,51 its Log. is 2.171756

Log. of R multiplied with t is 0,925596 } Add
The Multiplic. 0,01706 its Log. 8.231970 }
From 1ſt Log. ſubſtr. this Log. 9.157575
There remains the Logarith. 3.014181 of 1033,18

Viz. 1033 *l.* 3 *s.* 7 *d.* is the preſent Worth found by the Logarithms, *Caſe* 2. When P, t, and R, are given; to find U. That is, To find what Annuity, or Leaſe, *&c.* may be purchaſed for any propoſed Sum; to continue any aſſign'd Time, at any given Rate of Intereſt.

Rule.

Firſt, Multiply the Logarithm of R with the Time, and find the Number which belongs to the Product, and call that Number x, (as in the laſt Section.)

Again, Multiply the ſame Log. of R with the Time more 1, *and find the Number which belongs to that Product. Then multiply the Difference of theſe two Numbers with the propoſed preſent Worth, and their Product will be a Dividend.*

And the firſt Number called x, *being made leſs by* 1, *will be the Diviſer; by which if the aforeſaid Dividend be divided, the Quotient will ſhew the Annuity required for Yearly Payments.*

Example 1. For Yearly Rents.

What Annuity, or Yearly Rent, to continue twenty-one Years, may be purchaſed for 3205 l. 5 s. *at the Rate of Five* per Cent. per Annum?

Here is given $P = 3205{,}25$, $t = 21$, and $R = 1{,}05$; to find U.

$$\left.\begin{array}{r}\text{Firſt } R = 1{,}05 \text{ its Log. is } 0.021189\\ \text{And } \underline{t = \quad 21}\end{array}\right\}\text{Multiply}$$

The Product is the Log. 0.444969 of 2,7859 call'd x.

$$\left.\begin{array}{r}\text{Again, } R = 1{,}05 \text{ its Log. is } 0.021189\\ \text{And } \underline{t + 1 = \quad 22}\end{array}\right\}\text{Multiply}$$

Their Product is the Logar. 0.466158 of 2,9252

Then $2{,}9252 - 2{,}7859 = 0{,}1393$ the Multiplicator.

Next, $P = 3205{,}25 \times 0{,}1393$ gives 446,4913 for the Dividend.

And the Number call'd x $2{,}7859 - 1 = 1{,}7859$ is the Diviſor.

Then $1{,}7859)\,446{,}4913\,(250\,l. = U$ the Annuity required, for Yearly Payments.

Or having once found the two firſt Numbers, and by them form'd the Multiplicator and Diviſor, *as above*, the

Multiplication and Diviſion may be perform'd by Logarithms,

$$\left.\begin{array}{lr}\text{Thus, } \mathcal{P} = 3205,25 \text{ its Log.} & 3.505861 \\ \text{Multiplic. } 0,1393 \text{ its Log.} & 9.143951\end{array}\right\}\text{Add}$$

$$\left.\begin{array}{lr}\text{Sum} & 2.649812 \\ \text{The Diviſ. } 1,7859 \text{ its Log.} & 0.251857\end{array}\right\}\text{Subſtract}$$

Remains the Logarithm 2.397955 of $250 = U$. *Viz.* 250 *l.* will be the Yearly Annuity, as before : But this Caſe is more univerſally reſolved by the following Rule.

Rule 2.

Firſt, Multiply the Log. of R the Amount of 1 l. *with the Time, and find the Number which belongs to that Product, which call* x, (as before.)

Next, Multiply the propoſed preſent Worth with R — 1, viz. *with the Ratio of the given Rate, and multiply their Product with the firſt Number, called* x.

Then divide the laſt Product by the Number called x, *made leſs by* 1, *and the Quotient will ſhew the Annuity, or Rent for Yearly Payments,* (as before.)

As for inſtance in the laſt Example, wherein $\mathcal{P} = 3205,25$ $t = 21$, and $R = 1,05$; to find U.

$$\left.\begin{array}{lr}\text{Then } R = 1,05 \text{ its Log.} & 0,021189 \\ \text{And } t = & 21\end{array}\right\}\text{Multiply}$$

The Product is the Log. 0.444969 of 2,7859 call'd x.
Next, $R - 1 = 0,05$; and $\mathcal{P} = 3205,25 \times 0,05 = 160,2625$
Again, $160,2635 \times 2,7859 = 446,753$ the Dividend.
And $2,7859 - 1 = 1,7859$ for the Diviſor.
Then 1,7859)446,753(250 $= U =$ 250 *l.* the Annuity, or Rent for Yearly Payments.

The ſame may be perform'd by Logarithms.

Thus, If $R =$ 1,e5, and $t = 21$; then 2,7859 the Number call'd x will be found as before.

$$\left.\begin{array}{lr}\text{And if } \mathcal{P} = 3205,25 \text{ its Log. is} & 3.505861 \\ R - 1 = 0,05 \text{ its Log. is} & 8.698970 \\ \text{Numb. call'd } x \text{ } 2,7859 \text{ its Log. is} & 0.444969\end{array}\right\}\text{Add}$$

$$\left.\begin{array}{lr}\text{Sum} & 2.649800 \\ \text{Numb. } x - 1 = 1,7859 \text{ its Log.} & 0.251857\end{array}\right\}\text{Subſtr.}$$

There remains the Logarithm 2.397943 of 250

That is, 250 *l.* will be the Rent for Yearly Payments, (as before.)

But if the Annuity, or Rent, *&c.* is to be paid either *Half-yearly*, or *Quarterly*, &c.

Then, *Inſtead of multiplying the propoſed preſent Worth of the Annuity, with* $R - 1$ (*viz.* with the Ratio of the Rate, as in *Rule* 2.) *you muſt take ſuch a Part of the Log. of R, as the Time requires* (*viz.* the $\frac{1}{2}$, the $\frac{1}{4}$, or $\frac{1}{12}$, *&c.*) *and its Number made leſs by* 1, *muſt be the Multiplicator; and then proceed by* Rule 2. *as in the following Examples.*

Example 2. In Half-yearly Payments.

What Annuity, or Rent, to be paid every Half-Year, and to continue 21 *Years, may be purchaſed for the Sum of* 3244 l. 5 s. *at the Rate of* 5 per Cent. per Annum?

In this Queſtion there is given $P = 3244{,}25$, $t = 21$, and $R = 1{,}05$; to find U, the Half-yearly Rent.

$$\text{Thus } R = 1{,}05 \text{ its } \left.\begin{array}{r} \text{Log. } 0.021189 \\ \text{And } t = \underline{\qquad 21} \end{array}\right\}\text{Multiply}$$

The Product is the Log. 0.444969 of 2,7859 call'd x.

Then $2{,}7859 - 1 = 1{,}7859$ the Diviſor.

So far the Work is the ſame as for Yearly Payments.

Next, $R = 1{,}05$ the $\frac{1}{2}$ of its Log. is 0.010594 whoſe Number is 1,0247; and $1{,}0247 - 1 = 0{,}0247$ the Multiplicator inſtead of $0{,}05 = R - 1$.

Then $P = 3244{,}25 \times 0{,}0247$ the Product is 80,133; and $80{,}133 \times 2{,}7859$ the Number call'd x, the Product is 223,2432 for the Dividend.

Then $x - 1 = 1{,}7859)\ 223{,}2432\ (125 = U$.

That is, 125 *l.* will be the Half-yearly Annuity, or Rent, as was required by the Queſtion.

Or having once found the Number call'd x, and taken 1 from it for the Diviſor; and the Multiplicator uſed inſtead of $R - 1$, as before: then the Multiplication and Diviſion may be perform'd by Logarithms, thus:

$R = 1,05$ its Log. × with t is 0.444969
And $P = 3244,25$ its Log. is 3.511114 } Add
Multiplicator 0,0247 its Log. is 8.392697
Sum 2.348780 } Subſtract
The Diviſ. is 1,7859 its Log. is 0.251857
There remains the Logarith. 2.096923 of 125 = U. *&c.*

Example 3. In Quarterly Payments.

What Annuity, or Rent, to be paid every Quarter of a Year, and to continue 31 *Years and a half, may be purchaſed for* 1033 l. 3 s. 8 d. *at the Rate of* 7 per Cent. per Annum?

Here is given $P = 1033,185$; $t = 31,5$; and $R = 1,07$; to find U, the Quarterly Payments.

Firſt $R = 1,07$ its Log. is 0.029384
And $t =$ 31,5 } Multiply
The Product is the Log. 0.925596 of 8,4255 call'd x.
And $8,4255 - 1 = 7,4255$ for the Diviſor.

Again, $R = 1,07$ the ¼ of its Log. is 0,007346 whoſe Number is 1,01706; and $1,01706 - 1 = 0,01706$ for the Multiplicator, inſtead of $0,05 = R - 1$.

Then $P = 1033,185 \times 0,01706$ the Product is 17,6261
And $17,6261 \times 8,4255$ the Number call'd x, the Product will be 148,5087 the Dividend.

Then 7,4255) 148,5087 (20 $= U$; that is 20 *l.* will be the Quarterly Annuity, or Rent, as was required.

The ſame may be found by Logarithms, as in the laſt Example of Half-yearly Rents; which I ſuppoſe is needleſs to be ſet down.

Caſe 3. When U, P, and R are given; to find t.

That is, When any Annuity, with its propoſed Value or Purchaſe, and the Rate of Intereſt *per Cent.* &c. are given; thence to find how long the Purchaſer ought to enjoy it.

Rule.

Firſt, Multiply the propoſed Value or preſent Worth of the Annuity, with $R - 1$, (*viz.* with the Ratio of the given Rate of Intereſt) *and ſubſtract the Product from the Annuity, the Remainder will be the Diviſor.*

Next, Divide the Annuity by that Diviſor, and find the Log. of the Quotient; then if that Log. be divided by the Log. of R, the Amount of 1 l. *the Quotient will ſhew the Time, for Yearly Payments.*

Example 1. For Yearly Rents.

Suppoſe 3205 l. 5 s. *were lent upon an Annuity of* 250 l. per Annum, *in what Time would it pay off that Debt, allowing the Creditor* 5 per Cent. per Annum?

In this Queſtion there is given $P = 3205{,}25$; $U = 250$; and $R = 1{,}05$; to find t.

$$\left.\begin{array}{r} \text{First } P = 3205{,}25 \\ R - 1 = 0{,}05 \end{array}\right\}\text{Multiply}$$

And $U = 250 - 160{,}2625$ leaves $89{,}7375$ the Diviſor.

Next, 89,7375) 250,0000 (2,7859 whoſe Logarithm is 0.444969.

Then $R = 1{,}05$ its Log. is 0.021189) 0.444969 (21 = t.

That is 21 Years is the Time required in the Queſtion.

The ſame may be perform'd by Logarithms, thus:

$$\left.\begin{array}{l} \text{If } P = 3205{,}25 \text{ its Log. is } 3.505863 \\ \text{And } R - 1 = 0{,}05 \text{ its Log. } 8.698970 \end{array}\right\}\text{Add}$$

The Sum is the Logarith. 2.204833 of 160,2625

$$\left.\begin{array}{l} \text{Next, If } U = 250 \text{ its Log. is } 2.397940 \\ 250 - 160{,}2625 = 89{,}7375 \text{ Log. } 1.952972 \end{array}\right\}\text{Subſtract}$$

$R = 1{,}05$ its Log. is 0.021189) 0.444968 (21 = t.

Viz. 21 Years is the Time, if the Payments are Yearly.

But if the Annuity, or Rent, is to be paid, either by Half-yearly or Quarterly Payments, &c.

Then { *Inſtead of multiplying the propoſed preſent Worth of the Annuity with* R − 1 (as before) *you muſt take ſuch a part of the Logar. of R the Amount of* 1 l. *as the Time requires, and the Number anſwering to that part, being made leſs by* 1, *will be the Multiplicator; and then you may proceed on as before, according to the Rule.* }

Example 2. For Half-yearly Rents.

Admit a Man ſhould give 3244 l. 5 s. *for an Annuity, or clear Rent of* 250 l. per Annum, *to be paid Half-yearly,* (*viz.* 125 *l.* every Half-year;) *how long muſt he enjoy that Annuity, to be allow'd Five* per Cent. &c.)

Here is given $U = 125$, $P = 3244{,}25$, and $R = 1{,}05$; to find t, the Time or Number of Years.

Firſt $R = 1{,}05$ the $\frac{1}{2}$ of its Log. is 0.010594 whoſe Number is 1,0247; and $1{,}0247 - 1 = 0{,}0247$ the Multiplicator inſtead of $0{,}05 = R - 1$.

Then $P = 3244{,}25 \times 0{,}0247 = 80{,}1329$.

And $U = 125 - 80{,}1329$ leaves 44,8671 the Diviſ.

Next 44,8671) 125,0000 (2,786 whoſe Logarithm is 0,444981 for the Dividend.

Then $R = 1{,}05$ its Log. is 0.021189) 0.444981 (21 $= t$

That is, 21 Years is the Time that is required in the Queſtion.

The ſame perform'd by Logarithms:

Suppoſe $R = 1{,}05$ then the $\frac{1}{2}$ of its Log. 0.010594, whoſe Number is 1,0247; and $1{,}0247 - 1 = 0{,}0247$ the Multiplicator inſtead of $R - 1$, as before.

$$\left.\begin{array}{ll}\text{Then } P = 3244{,}25 \text{ its Log. is} & 3.511114 \\ \text{The Multiplic. } 0{,}0247 \text{ its Log.} & \underline{8.392697}\end{array}\right\}\text{Add}$$

Their Sum is the Logarithm 1.903811 of 80,133

$$\left.\begin{array}{ll}\text{Next, } U = 125 \text{ its Logar. is} & 2.096910 \\ \text{And } 125 - 80{,}133 = 44{,}867 \text{ its Log.} & \underline{1.651927}\end{array}\right\}\text{Subſtr.}$$

Then $R = 1{,}05$ its Log. is 0.021189) 0.444983 (21 $= t$.

Viz. 21 Years, *&c.* as before.

Example 3. In Quarterly Payments.

Suppoſe one ſhould give 1033 l. 3 s. 8 d. *for an Annuity of* 80 l. per Annum, *to be paid Quarterly,* (*viz.* to have 20 *l.* every Quarter of a Year;) *how long may he enjoy it, to be allow'd Seven* per Cent. per Annum?

In this Queſtion there is given $U = 20$, $P = 1033{,}185$, and $R = 1{,}07$; to find t, the Time or Number of Years.

Thus, $R = 1{,}07$ the $\frac{1}{4}$ of its Log. is 0.007346 whoſe Number is 1,01706; and $1{,}01706 - 1 = 0{,}01706$ for the Multiplicator, to be uſed inſtead of $0{,}07 = R - 1$.

Then $P = 1033{,}185 \times 0{,}01706 = 17{,}6261$; which being taken from $U = 20$, leaves 2,3739 the Diviſor.

Then 2.3739) 20,0000 (8425 its Log. 0.925570.

Laſtly, $R = 1,07$ its Log. 0.029384) 0.925570(31,5 = t.
Viz. the Time required will be 31 Years and a half.

Which may alſo be found by Logarithms,
Thus, If $R = 1,07$ the $\frac{1}{4}$ of its Log. is 0.007346, whoſe Number is 1,01706; and 1,01706 — 1.
Leaves 0,01706 for the Multiplicator uſed inſtead of $R - 1$.

Then $\mathcal{P} = 1033,185$ its Log. is 3.014181 }
Multiplicator 0,01706 its Log. 8.231979 } Add
Their Sum is the Logarithm 1.246160 of 176262
And if $U = 20$, then 20 — 17,6262 = 2,3738 the Diviſor.

Again, $U = 20$ its Logar. is 1.301030 }
Diviſor 2,3738 its Logarithm is 0.375444 } Subſtract
$R = 1,07$ its Log. is 0.029384) 0.925586(31,5 = t, &c.

Caſe 4. By having U, t, and $\mathcal{P}$, given; to find R.
That is, When any Annuity, or Rent *per Annum*, with the Time of its Continuance, and its propoſed preſent Worth for that Time are given; thence to find what Rate of Intereſt *per Cent.* &c. is allow'd the Purchaſer.

This Caſe is ſo difficult, that the Ingenious *Michael Dary*, in his Treatiſe of *Intereſt* epitomiz'd, *pag.* 7. ſets it down as very troubleſome to be reſolved, in regard (*ſays he*) that R (*the thing ſought*) cannot be brought to one Side of the Equation ſolely, and the other Side clear. But I rather think that the true Cauſe which rendred a perfect Solution of this Caſe ſo troubleſome to him (*or indeed impoſſible at that time*) was becauſe the reſolving of ſuch high Equations as this Caſe requires, was not in his Time ſo well known as now it is.

For the Equation, by which the true Value of R may be found, is this;

$$Viz.\ \frac{U}{\mathcal{P}} = \frac{U}{\mathcal{P}} R^t + R^t - R^{t+1}$$

Which I reduce to the following Equation, as being much better for our preſent Purpoſe;

$$Viz.\ U = UR^t - \mathcal{P}R^t \times R - 1.$$

Either of theſe Equations is more Adfected than that in *pag.* 174. and requires an Algebraical Solution to diſcover the true Value of R; and can be no otherwiſe found here, but only by the ſame manner of Approachment, as before in the fourth *Caſe* of the laſt *Section*: However, in order to render that Approximation as eaſy and ſhort as poſſibly I can, obſerve the following Directions.

Viz.

Firſt, Aſſume ſuch a Number for the Value of R, *as you gueſs may be the Amount of* 1 l. *at the Rate of the Intereſt required* (as before in *pag.* 144). *Then multiply the Log. of that ſuppoſed R with the Time, and find the Number which belongs to that Product, and multiply it with the Annuity: Call the Product* z.

Next, Multiply the ſame Number with the propoſed preſent Worth, and multiply their Product with $R - 1$, (viz. with the Ratio of the ſuppoſed Rate.)

Then ſubſtract the laſt Product from the Product call'd z. *If the Remainder be equal to* (*viz.* be the ſame with) *the Annuity, you have hit right upon the true Value of R: But if it be leſs than the Annuity, then is the Number of R taken too big: And on the contrary, if it be more than the Annuity, that R is too little; and another Tryal muſt be made accordingly.*

Example 1. In Yearly Rents.

Suppoſe there were paid 3205 l. 5 s. *for an Annuity, or a Leaſe of* 250 l. per Annum, *to continue* 21 *Years; what Rate of Intereſt* per Cent. &c. *is allow'd to the Purchaſer?*

In this Queſtion there is given $U = 250$, $t = 21$, and $P = 3205{,}25$; to find R, the Amount of 1 *l.*

Firſt, I will ſuppoſe R to be 1,06; that is, the Amount of 1 *l.* at the Rate of 6 *per Cent.*

$$\left.\begin{array}{r} \text{Then if } R = 1{,}06 \text{ its Log. is } 0.025306 \\ \text{And } t = \qquad 21 \end{array}\right\}\text{Multiply}$$

The Product is the Logarith. 0.531426 of 3,3996

Then $U = 250 \times 3{,}3996$ the Product is 849,9 which is the Number call'd z.

Next, $P = 3205{,}25 \times 3{,}3996$ the Product is 10896,5679 And $10896{,}5679 \times 0{,}06$ (viz. *with* $R - 1$) the Product will be 653,7941. Then if this Number be ſubſtracted from 849,9 the Number call'd z, there will remain 196,1059 which is leſs than 250 the Annuity: And therefore I conclude that the ſuppoſed $R = 1{,}06$ is too big.

Again, For a ſecond Tryal, I will ſuppoſe $R=1{,}05$

$$\left.\begin{array}{r}\text{Then if } R=1{,}05 \text{ its Log. is } 0.021189\\ \text{And } t= \qquad 21\end{array}\right\}\text{Multiply}$$

The Product will be the Log. 0.444969 of 2.7859

And $250 \times 2{,}7859 = 696{,}475$ the Number call'd z.

Next $3205{,}25 \times 2{,}7859 = 8929{,}5$, which being multiplied with $0{,}05 = R-1$, the Product is 446,475.

Then z, $696{,}475 - 446{,}475 = 250{,}000$; which is the very ſame with the Annuity: Whence I conclude that the true Value of R is 1,05; and $R-1=0{,}05$ will be the Ratio of the Rate of Intereſt ſought.

Then it will be, As 1 : is to 0,05 :: ſo is 100 : to 5, the Rate of Intereſt *per Cent.* as was required for Yearly Payments.

Now theſe Trials for finding the true Value of R, may be perform'd by Logarithms with a very little trouble: As for inſtance, in this laſt Example,

Wherein $U=250$, $P=3205{,}25$, and $t=21$; to find R

$$\left.\begin{array}{r}\text{Suppoſe } R=1{,}06 \text{ its Log. is } 0.025306\\ \text{And } t= \qquad 21\end{array}\right\}\text{Multiply}$$

$$\left.\begin{array}{r}\text{Sum } 0.531426\\ \text{Next } U=250, \text{ its Log. is } 2.397940\end{array}\right\}\text{Add}$$

The Sum is the Logarithm 2.929366 of 849,9 the Number call'd z.

$$\left.\begin{array}{r}\text{Again, } R=1{,}06 \text{ its Log.} \times \text{with } t, \text{ is } 0.531426\\ \text{And } P=3205{,}25 \text{ its Log. is } 3.505863\\ R-1=0{,}06 \text{ its Log. is } 8.778151\end{array}\right\}\text{Add}$$

The Sum is the Logarith. 2.815440 of 653,79

Then z, $849{,}9 - 653{,}79 = 196{,}11$ which is leſs than 250 the Annuity: whence it appears that $R=1{,}06$ is too big.

$$\left.\begin{array}{r}2ly. \text{ Let } R=1{,}05 \text{ its Log. } 0.021189\\ \text{And } t= \qquad 21\end{array}\right\}\text{Multiply}$$

$$\left.\begin{array}{r}\text{Sum } 0.444969\\ \text{Next, } U=250 \text{ its Log. } 2.397940\end{array}\right\}\text{Add}$$

The Sum is the Logar. 2.842909 of 696,48 call'd z.

$$\left.\begin{array}{r}\text{Again, } P=3205{,}25 \text{ its Log. is } 3.505863\\ R=1{,}05 \text{ its Log.} \times \text{with } t, \text{ is } 0.444969\\ \text{And } R-1=0{,}05 \text{ its Log. is } 8.698970\end{array}\right\}\text{Add}$$

Their Sum is the Logarithm 2.649802 of 446,48

Then *z*, 696,48 — 446,48 = 250, which is juſt the ſame with the Annuity; therefore the true Value of *R* is 1,05 *&c.* as before.

But if the Annuity, or Rent, is to be paid either Half-yearly, or Quarterly, *&c.*

$$\text{Then,}\left\{\begin{array}{l}\textit{Instead of multiplying with } R-1\text{, viz. }\textit{with}\\ \textit{the Ratio of the supposed Rate of Interest}\text{ (as}\\ \text{above) }\textit{you must take such a part of the Log.}\\ \textit{of } R \text{ (the assumed Amount of 1 } l.\text{) }\textit{as the}\\ \textit{Time requires, and make its Number the Mul-}\\ \textit{tiplicator}\text{, \&c. }\textit{as in the following Examples.}\end{array}\right.$$

Example 2. In Half-yearly Payments.

Admit 3244 l. 5 s. *were given for an Annuity, or Leaſe of* 250 l. per Annum, *to be paid Half-yearly,* (*viz.* 125 l. every Half-year) *and to continue* 21 *Years*; *what Rate of Intereſt* per Cent. *is allow'd the Purchaſer?*

In this Example there is given $U = 125$, $t = 21$, and $\mathcal{P} = 3244{,}25$; to find *R*, the Amount of 1 *l.*

$$\left.\begin{array}{rr}\text{First, I suppose } R = 1{,}06 \text{ its Log.} & 0{,}025306\\ \text{And } t = & \underline{\qquad 21}\end{array}\right\}\text{Multiply}$$

Their Product is the Logarith. 0.531426 of 3,3996

Then $U = 125 \times 3{,}3996$ the Product is 424,95 call'd *z*.

Next, $\mathcal{P} = 3244{,}25 \times 3{,}3996$ the Product is 11029,1523

Again, If $R = 1{,}06$ the $\frac{1}{2}$ of its Log. is 0.012653 whoſe Number is 1,02956; and $1{,}02956 - 1 = 0{,}02956$ the Multiplicator inſtead of $0{,}06 = R - 1$.

Then $11029{,}1523 \times 0{,}02956 = 326{,}0217$ which being ſubſtracted from 424,95 call'd *z*, there will remain 98,9283 which is leſs than 125 the Half-yearly Annuity; whence I conclude that the ſuppoſed $R = 1{,}06$ is taken too big.

And therefore for a ſecond Tryal I will ſuppoſe $R = 1{,}05$ and proceed by Logarithms only:

$$\left.\begin{array}{rr}\text{Then if } R = 1{,}05 \text{ its Log.} & 0.021189\\ \text{And } t = & \underline{\qquad 21}\end{array}\right\}\text{Multiply}$$

$$\left.\begin{array}{rr}\text{Sum} & 0.444959\\ \text{Next, } U = 125 \text{ its Log. is} & \underline{2.096910}\end{array}\right\}\text{Add}$$

Their Sum is the Logar. 2.541879 of 348,24 call'd *z*.

Again, if $R = 1{,}05$ the $\frac{1}{2}$ of its Log. is 0.010594, whoſe Number is $1{,}0247 - 1$, leaves 0,0247 the Multiplicator.

Then

$$\left.\begin{array}{ll} \text{Then } P = 3244{,}25 \text{ its Log. is} & 3.511114 \\ \text{And } R = 1{,}05 \text{ its Log.} \times \text{with } t, \text{ is} & 0.444969 \\ \text{Multiplicator } 0{,}0247 \text{ its Log. is} & \underline{8.392697} \end{array}\right\} \text{Add}$$

Their Sum is the Logarith. 2.348780 of 223,24

Then z, 348,24 — 223,24 = 125 which is the very ſame with 125 the Half-yearly Annuity; and therefore I conclude that $R = 1{,}05$ and $R - 1 = 0{,}05$ is the Ratio of the Rate of Intereſt ſought.

Conſequently, as 1 : is to 0,05 : : ſo is 100 : to 5, the true Rate of Intereſt *per Cent.* &c.

I ſhall cloſe up this *Caſe* with an *Example* of finding the Rate of Intereſt, that is allow'd to the Purchaſers of the *Parliamentary* Annuities, and reſolve it by Logarithms only.

Example 3. Of Quarterly Payments.

There is 1600 l. *paid for an Annuity of* 100 l. per Ann. *to continue* 99 *Years, and to be paid Quarterly,* viz. 25 l. *every Quarter of a Year during that Time: 'Tis required to find what Rate of Intereſt* per Cent. *is allow'd the Purchaſer.*

In this Queſtion there is given $U = 25$, $t = 99$, and $P = 1600$; to find R, &c.

Firſt, I will here ſuppoſe the Rate of Intereſt ſought to be 6 *per Cent.* for the Reaſons mention'd in *pag.* 159.

$$\left.\begin{array}{rr} \text{Then if } R = 1{,}06 \text{ its Log.} & 0.025306 \\ \text{And } t = & \underline{99} \end{array}\right\} \text{Multiply}$$

$$\left.\begin{array}{rr} \text{The Product is} & 2.505294 \\ \text{Next, } U = 25 \text{ its Log. is} & \underline{1.397940} \end{array}\right\} \text{Add}$$

The Sum is the Logarith. 8002,6 of 3.903234 called z.

Again, if $R = 1{,}06$ the ¼ of its Log. is 0.006326, whoſe Number is 1,01467 leſs 1, leaves 0.01467 the Multiplicator.

$$\left.\begin{array}{ll} \text{Then } P = 1600 \text{ its Log. is} & 3.204120 \\ \text{The Log. of } R \text{ multipl. with } t & 2.505294 \\ \text{Multiplicator } 0.01467 \text{ its Log.} & \underline{8.166430} \end{array}\right\} \text{Add}$$

Their Sum is the Logarith. 3.875844 of 7513,5

Which being taken from 8002,6 = z, leaves 489,1; which is more than 25 = U; and therefore the Value of R is taken too little.

And for a ſecond Trial I do aſſume *R* = 1,064

Then if *R* = 1,064 its Log. 0.026942 } Multiply
And *t* = 9

The Product is 2.667258 } Add
Next, *U* = 25 its Log. is 1.397940

The Sum is the Logarith. 4.065198 of 11619,8 = *z*.

Again, the $\frac{1}{4}$ of the Log. of *R* is 0.006735 its Number is 1,01563; and 1,01563 − 1 = 0.01563 the Multiplicator.

Then *P* = 1600 its Log. 3.204120 }
Log. of *R* × with *t*, is 2.667258 } Add
Multiplic. 0.01563 its Log. $\bar{8}$.193959 }

Their Sum is the Logar. 4.065337 of 11623,5; which is more than 11619,8 = *z*; and therefore the Value of this *R* = 1,064 is ſome ſmall matter too big: conſequently *R* is ſome Nnmber between 1,06 and 1,064; and is but a very little leſs than 1,064.

Let *R* = 1,0638 its Log. 0.026860 } Multiply
And *t* = 99

Product 2.659140 } Add
Next, *U* = 25 its Log. is 1.397940

The Sum is the Logar. 4.057080 of 11414,6 = *z*.

Again, *R* = 1,0638 the $\frac{1}{4}$ of its Log. is 0.006715; whoſe Number is 1,01558 leſs 1 is 0,01558 the Multiplicator.

Then *P* = 1600 its Log. is 3.204120 }
Log. of *R* multiplied with *t* 2.659140 } Add
Multiplicat. 0,01558 its Log. $\bar{8}$.192567 }

The Sum is the Logarithm 4.055827 of 1137,7

Then the Number call'd *z*, *viz.* 11414,6 — 11377 = 37,6 which is now a ſmall matter more than 25 = *U*.

Conſequently *R* = 1,0638 is a very ſmall matter too little, which I judge muſt be betwixt 1,0638 and 1,0639; and accordingly I make one other Trial with aſſuming the Value of *R* = 1,063855, and find it pretty near the Truth. Conſequently *R* — 1 = 0,063855 the Ratio of the Rate. And then it will be, As 1 : 0,063855 :: 100 : 6,3855 That is, *6 l.* 7 *s.* $8\frac{1}{2}$ *d.* will be very near the true Rate of Intereſt *per Cent.* as was required.

And for a Confirmation of the Truth thereof, let us take it for granted that *R* = 1,063855, as above; and let there be given the Quarterly Annuity, or *U* = 25 *l.* the Time, or *t* = 99 Years; to find *P*, the preſent Worth (*per Caſe* 1.) which we already know muſt be 1600 *l.* if the Value of *R* be truly found. Then,

If *R* = 1,063855 its Log. is 0.026882 } Multiply
And *t* = 99 }

Their Product is 2.661318 } Add
Next, *U* = 25 its Log. is 1.397940 }

The Sum is the Logarith. 4.059258 of 11461,94

From this 11461,94 ſubſtract the Quarterly Rent 25, there remains 11436,94 the Dividend.

Again, *R* = 1,063855 the ¼ of its Log. is 0.006720; whoſe Number is 1,015592 and 1,015592 − 1 = 0.015592 the Multiplicator inſtead of 0,063855 = *R* − 1.

The Dividend 11436,94 its Log. is 4.058312

Log. of *R* multiplied with *t*, is 2.661318 } Add
Multiplicator 0,015592 its Log. is $\bar{8}$.192902 }

From the upper Log. ſubſtract this Log. 0.854220 Sum.

There remains the Logarith 3.204092 of (1599,9 = *P*.

That is, the preſent Worth thus found is 1599 *l.* 18 *s.* which wants but 2 *s.* of 1600 *l.* the true preſent Worth. Whence we may be aſſured, that the Value of *R*, and conſequently the Rate of Intereſt *per Cent.* found as above, is very near the Truth, for Quarterly Payments. But ſuppoſing thoſe Annuities were to be paid but once a Year, then the Rate of Intereſt will be only 6 *l.* 5 *s. per Cent.* or very near it, as may be eaſily tried, either by finding the preſent Worth, as in *Example* 1. *pag.* 180. or by finding the Annuity, as in *Example* 1. *pag.* 184, &c.

Thus you have a full Solution at large to the eight Caſes, which include all the Varieties that can be propoſed, either relating to the Arrears, or about the purchaſing of Annuities, and the taking of ſuch Leaſes as are either in preſent Poſſeſſion when the Contract is made, or are to be immediately enter'd upon. But for ſuch Queſtions, as relate to Annuities, and taking ſuch Leaſes, &c. as are in Reverſion, they require a further Conſideration.

Sect. 4. *To find the Preſent Worth of Annuities, or Leaſes,* &c. *in Reverſion at Compound Intereſt.*

I Have already touch'd upon this Part of the Work, *pag.* 156, &c. as it may be perform'd at *Simple Intereſt*; but ſhall here handle it more fully in all its uſeful Caſes. For altho' they only depend upon the due Application of the Rules already laid down; yet it will be convenient to ſhew how thoſe Rules are to be truly applied, according as the particular Caſes require.

Caſe 1. When the Annuity, or Yearly Rent, with the Time of its Continuance in Poſſeſſion, and the Time of the Reverſion, (viz. *the Time it's not to be in Poſſeſſion*) are given; to find its preſent Worth, at any aſſign'd Rate of Intereſt *per Cent.*

The Solution of this Caſe requires two diſtinct Operations, which muſt be thus perform'd.

Firſt { *Find the Value or Preſent Worth of the propoſed Annuity, or Rent, for the given Time of its Continuance, as if it were immediately to be entered upon.*

Then { *Find what Principal, or Sum, being forborn at Intereſt, during the Time of the Reverſion, would amount to, or raiſe the aforeſaid Value; and that Principal will be the Preſent Worth of the propoſed Annuity,* &c. *in Reverſion.*

Example in Yearly Rents.

There is the Reverſion of a Leaſe of 175 l. per Annum, *to be lett for eleven Years, which are not to commence until after nine Years are expired; 'tis required to find the Preſent Worth of that Leaſe, allowing the Rate of Six* per Cent. *Intereſt to the Purchaſer.*

Or thus:

There is one has nine Years to come in a Leaſe of 175 l. per Annum, *and he's deſirous to enlarge his Time eleven Years more* (*viz.* to enjoy his Leaſe twenty Years yet to come;) *what Sum muſt he give in ready Money for that Purchaſe, to be allow'd the Rate of* 6 per Cent. &c.

In the firſt Work of this Example there is given $U = 175$, $t = 11$, and $R = 1,06$; to find P, the Preſent Worth, as in *Caſe* 1. *pag.* 180.

Thus $R = 1,06$ its Log. is 0.025306 } Multiply
And $t =$ 11

The Product is 0.278366 } Add
Again $U = 175$ its Log. is 2.243038

The Sum is the Logarithm 2.521404 of 332,203

Then from 332,203 ſubſtract the Rent 175, and there remains 157,203 its Log. is 2.196459

The firſt Log. × with $t = 11$ is 0.278366 } Add
And $R - 1 = 0,06$ its Log. is $\bar{8}$.778151

From up. Log. ſubſt. this Log. 9.056517

There remains the Logar. 3.139942 of $1380,2 = P$, the Preſent Worth, ſuppoſing the Rent were to be immediately enter'd upon; but becauſe it is not ſo, therefore, for the ſecond Part of the Work there is given $A = 1380,2$, $t = 9$, and $R = 1,06$; to find P, the Principal, as in *Caſe* 2. *pag.* 162.

Thus $A = 1380,2$ its Log. is 3.139943 } Subſtract
$R = 1,06$ its Log. × $9 = t$, is 0,227754

And there remains the Log. 2.912189 of $816,937 = P$. *Viz.* 816 *l.* 18 *s.* 9 *d.* will be the true Preſent Worth of that Leaſe in Reverſion, as was required, *viz.* for Yearly Rents.

But if the Rents are to be paid either *Half-yearly*, or *Quarterly*, &c. then the firſt part of the Work muſt be found as in their reſpective Examples, *viz.* for Half-yearly Payments, it muſt be found as in the Example, *pag.* 181. and for Quarterly Payments, as in the Example, *pag.* 182. And then proceed on as above.

Caſe 2. To find what Annuity, or Yearly Rent in Reverſion, may be purchaſed for any propoſed Sum, at any aſſign'd Rate of Intereſt *per Cent.* when the Time the Annuity is not to be enter'd upon; and the Time of its Continuance, are both given.

The Solution of this Caſe is but the Reverſe of the laſt, and doth alſo require two diſtinct Operations.

First { *Find what the Sum propoſed to be laid out in the Reverſion would amount to, ſuppoſing it were forborn at the given Rate of Intereſt, during the Time that the Annuity is not to be in Poſſeſſion.*

Then, { *Find what Annuity, or Yearly Rent, that Amount will purchaſe, and that will be the Anſwer required.*

Example in Yearly Rents.

What Annuity, or Yearly Rent, to commence nine Years hence, and then to continue eleven Years after, may be purchaſed for 816 l. 18 s. 9 *d. ready Money; at the Rate of* 6 per Cent. &c.

In the firſt Work of this Queſtion there is given, $P=816{,}937$, $R=1{,}06$, and $t=9$ (*the Time that the Annuity is not to be enter'd upon;*) to find *A*, as in *Caſe* 1. *page* 162.

Thus $R=1{,}06$ its Log. 0.025306 } Multiply
And $t=$ 9 }

The Product is 0.227754 } Add
Next $P=816{,}937$ its Log. 2.912189 }

The Sum is the Logar. 3.139943 of 1380,2 $=A$ the Amount of 816 *l.* 18 *s.* 9 *d.* which we muſt now call *P*, for the ſecond Part of the Work; wherein there is given $P=1380{,}2$, $R=1{,}06$, and $t=11$, (*the Time that the Annuity is to be enjoy'd*) to find *U*, as in *Caſe* 2. *pag.* 183.

Thus $R=1{,}06$ its Log. is 0.025306 } Multiply
And $t=$ 11 }

The Product is the Log. 0.278366 of 1,8983
And $1{,}8983-1=0{,}8983$ is the Diviſor.

Again, $R=1{,}06$ its Log. is 0.025306 } Multiply
And $t+1=$ 12 }

Their Product is the Logar. 0.303672 of 2,0122
And $2{,}0122-1{,}8983=0.1139$ the Multiplicator,

Then $P=1380{,}2$ its Log. is 3.139943 } Add
Multiplicator 0,1139 its Log. 9.056524 }

Sum 2.196467 } Subſtract
The Diviſor 0,8983 its Log. 9.953421 }

There remains the Logar. 2.243046 of 175 $=U$.

That is, 175 *l. per Annum* is the Rent, or Annuity required by the Queſtion.

But if the Rents are to be paid every *Half-year*, or every *Quarter*, &c. then the ſecond Part of the Work muſt be perform'd as in their reſpective Examples; *viz.* For Half-yearly Payments, U muſt be found, as in the Example, *pag.* 185. and for Quarterly Payments, as in the Example, *pag.* 186, &c.

Caſe 3. Any Annuity, or Yearly Rent, and the Sum that's propoſed to be paid for a Leaſe of it, to commence after a certain Term or Number of Years are paſt, being given; thence to find how long that Leaſe ought to continue in the Poſſeſſion of the Purchaſer, allowing him any aſſign'd Rate of Intereſt *per Cent.*

This Caſe does alſo require two ſeparate Operations.

Viz. { *Firſt, To find what Sum the propoſed Money to be laid out in the Reverſion, would amount to,* &c. *as in the firſt Part of the laſt Caſe.*

Then { *Having the Annuity, with its preſent Worth or Value, and the Rate of Intereſt given; to find the Time,* &c. *as in* Caſe 3. page 187.

Example in Yearly Rents.

Suppoſe a Debt of 816 l. 18 s. 9d. *were propoſed to be paid off, by making over a Leaſe of* 175 l. per Annum *in Reverſion, that is not to be enter'd upon by the Creditor, until nine Years are paſt; the Queſtion is, How many Years the Creditor may enjoy that Leaſe, to have his Debt cleared, and to be allowed the Rate of 6* per Cent. per Annum.

For the firſt Work in this Queſtion, there is given $P = 816{,}937$, $R = 1{,}06$, and $t = 9$ (*the Time before the Leaſe is to commence*) to find A; which will be found $= 1380{,}2$ as in the firſt Part of the laſt Caſe; which we muſt here alſo call P, for the ſecond Part of the Work.

Then there will be given new $P = 1380{,}2$ $U = 175$ and $R = 1{,}06$; to find the $t =$ to the Time which the Leaſe is to be enjoyed: As in *Example* 1. *page* 187.

Thus

$$\left.\begin{array}{ll}\text{Thus } P=1380{,}2 \text{ its Log. is} & 3.139942 \\ R-1=0{,}06 \text{ its Log. is} & \underline{\bar{8}.778151}\end{array}\right\}\text{Add}$$

The Sum is the Logarithm 1.918093 of $82{,}812$.

$$\left.\begin{array}{ll}\text{Next } U=175 \text{ its Log. is} & 2.243038 \\ 175-82{,}812=92{,}188 \text{ its Log.} & \underline{1.964674}\end{array}\right\}\text{Subſtract.}$$

$R=1{,}06$ its Log. is $0.025306)0.278364(\,11=t$.

Viz. 11 Years will be the Time required by the Queſtion; ſuppoſing the Rents to be paid but once a Year.

But if the Rents are to be paid every Half-Year, then t, the Time the Leaſe is to be enjoy'd, may be found as in *Example, page* 187. And if Quarterly, then as in *Example, page* 189, *&c.*

Now theſe three Caſes include all the uſual (and uſeful) Queſtions that relate to purchaſing Annuities, or taking of Leaſes in Reverſion; and altho' I've made choice of ſhewing how to reſolve each of them by two ſeparate and diſtinct Operations, (being, as I preſume, the eaſieſt Way to be underſtood) yet they may be reſolved at one, by a due Reduction of the following Equation. For

Putting $\left\{\begin{array}{l} t = \text{the } \textit{Time} \text{ the } \textit{Leaſe}, \text{\&c. is to be enjoy'd;} \\ T = \text{the } \textit{Time} \text{ of its } \textit{Reverſion}\text{; then}\end{array}\right.$

it will be $UR^t - U = PR^T \times R^t \times R-1$.

And from hence the Value of R, *viz.* the Rate of Intereſt, may alſo be found; which I take to be a Work more of Curioſity, than of any great Uſe: For the Rate of Intereſt is always ſuppoſed to be known, it being one of the main or chief Conſiderations in all Bargains about taking or letting of Leaſes, *&c.* And therefore the finding of R in the Caſe of Reverſions, (or indeed in the two 4th Caſes of the two laſt Sections) is rather for a Proof of the Work, and to compleat the Rounds, than for any thing elſe. However, if any one will (out of his Curioſity) give himſelf the trouble of doing it, he may proceed by ſuch Approximation as in the aforeſaid Sections, having a due regard to the different Values of t, and T. But, for brevity ſake, I ſhall omit giving Examples thereof.

CHAP. IV.

Of Purchaſing Freehold or Real Eſtates, and how to eſtimate the Value of Annuities, and Leaſes for Lives, &c.

ALL *Freehold*, or *Real Eſtates*, are ſuppoſed to be purchaſed (or bought) to continue for ever; *viz.* without the Conſideration of any limited Time: Therefore the Buſineſs of computing the true Value or Worth of ſuch Eſtates (*according to Art*) is grounded upon a Rank, or Series, of Geometrical Proportionals, continually decreaſing *ad infinitum*; from whence the following Rules are deduced.

Sect. 1. *Of Purchaſing Freehold Eſtates.*

I Shall here make uſe of the ſame Letters to denote the ſeveral Parts of the Queſtion, as in the laſt Chapter, ſave only that *t*, the Time to limit the Number of Payments, is not concern'd.

Viz. U = the propoſed *Rent* of an *Eſtate*.
P = the *Purchaſe*, or *Worth* of that *Eſtate*.
R = the *Amount* of 1 *l.* at any Rate of *Intereſt*.

Then it will be $PR - P = U$.

This Equation admits of three different Caſes.

Caſe 1. The Annual Rent of any Freehold Eſtate being known; to find what it's worth in Ready Money, allowing the Purchaſer any aſſign'd Rate of Intereſt *per Cent.*

Rule. *Divide the propoſed Rent by the Ratio of the given Rate of Intereſt* (*viz.* by $R-1$) *and the Quotient will ſhew what the Eſtate is worth at that Rate of Intereſt.*

Example 1.

Suppoſe a Freehold Eſtate of 250 l. *Yearly Rent, were to be ſold; what is it worth, allowing the Buyer the Rate of* 6 per Cent. *Compound Intereſt for his Money?*

Here is given $U=250$, $R=1{,}06$; to find P.

Thus $R-1=0{,}06)\ 250{,}00\ (4166{,}6667=P$.

That is, 4166 *l.* 13 *s.* 4 *d.* is the Preſent Worth of that Eſtate; which is 16 Years and 8 Months Value, ſuppoſing the Rents to be paid but once a Year.

The ſame may be found by Logarithms,

Thus $U=250$ its Log. 2.397940 } Subſtract
And $R-1=0{,}06$ its Log. 8.778151 }
There remains the Log. 3.619789 of 4166,66 = P. *viz.* 4166 *l.* 13 *s.* 4 *d.* &c. as above.

Note, *Here it is ſuppoſed that the Purchaſer is to enter immediately into Poſſeſſion of the Eſtate: But if it be an Eſtate in Reverſion, that is, not to be entered upon until after ſome Term of Years are paſt; then you muſt compute its preſent Worth, as in* Caſe 1. pag. 196. *and ſo on in the following Caſes, compared with their reſpective Caſes in* Sect. 4. *of the laſt* Chapter.

If the Rents are to be paid either Half-yearly, or Quarterly, as moſt generally they are;

Then { *Inſtead of dividing the propoſed Rent by* R − 1, *as above, you muſt take ſuch a part of the Log. of* R *the Amount of* 1 l. *as the Times of Payment require; and the Number anſwering to that part, being made leſs by* 1, *muſt be made the Diviſor inſtead of* R − 1.

Example in Half-yearly Rents.

Suppoſe the aforeſaid 250 l. per Annum *were to be paid by Half-yearly Rents* (viz. 125 *l.* every Half-year) *what would that Eſtate be then worth, at the ſame Rate of Intereſt,* viz. 6 per Cent.? &c.

In this Example there is given $U=125$, and $R=1{,}06$; to find P. Thus,

Firſt, $R=1{,}06$ the $\frac{1}{2}$ of its Log. is 0.012653 whoſe Number is 1,029563; and 1,029563 − 1 = 0,029563 for the new Diviſor inſtead of $R-1$.

Then $U=125$ its Log. is 2.096910 } Subſtract
Diviſor 0,029563 its Log. 8.470748 }
There remains the Logar. 3.626162 of 4228,26 = P.

So that 4228 *l.* 5 *s.* is here the preſent Worth or Value of the ſame Eſtate, which is 61 *l.* 11 *s.* 4 *d.* more than its Worth for Yearly Rents.

Example 3. For Quarterly Rents.

Let it be required to find what the ſame Eſtate (viz. 250 *l.* per Annum) *is worth, when the Rents are to be paid Quarterly* (viz. 62 *l.* 10 *s.* every Quarter of a Year) *at 6* per Cent. *Intereſt*, &c. *as before.*

Here is given $U = 62,5$ $R = 1,06$ to find P.

Thus, $R = 1,06$ the $\frac{1}{4}$ of its Log. is 0,006326 whoſe Number is 1,014674 and $1,014674 - 1 = 0,014674$ the Diviſor inſtead of $R - 1$.

Then $U = 62,5$ its Log. is 1.795880 } Subſtract
Diviſ. 0,014674 its Logar. $\bar{8}.166548$ }

There remains the Log. 3.629332 of $4259,24 = P$.

Viz. 4259 *l.* 4 *s.* 9 ½ *d.* is now the Value, or Preſent Worth of the ſame Eſtate; which is ſomething above 17 Years Value.

Now from hence it plainly appears, that, although there is no ſuch thing as a limited Time conſidered in the purchaſing of Freehold Eſtates; yet there is, and ought to be, a due Reſpect had to the Times of their Rents being paid: for according to that, their Values are more or leſs worth.

Caſe 2. When any Sum of Money is propoſed to be laid out in a Freehold Eſtate; to find what Annual Rent that Sum will purchaſe, at any given Rate of Intereſt *per Cent.* &c.

Rule. { *Multiply the propoſed Sum to be laid out in the Eſtate, with* $R - 1$; viz. *with the Ratio of the given Rate of Intereſt* per Cent, *and the Product will ſhew the Yearly Rent required.*

Example in Yearly Rents.

Suppoſe 3560 l. *were propoſed to be laid out in the Purchaſe of a Freehold Eſtate; what annual Rent would it buy, allowing the Purchaſer but Four* per Cent. per Annum *Intereſt.*

In this Example there is given $P = 3560$, $R = 1{,}04$, to find U, the Yearly Rent.

$$\left.\begin{array}{ll}\text{Thus} & P = 3560\\ \text{And} & R - 1 = 0{,}04\end{array}\right\}\text{Multiply}$$

Product is $142{,}4 = 142$ *l.* 8 *s.* $= U$ the Yearly Rent required.

Or the ſame may be found by Logarithms.

$$\left.\begin{array}{lll}\text{Thus } P = 3560 & \text{its Log. is} & 3.551450\\ R - 1 = 0{,}04 & \text{its Log. is} & 8.602060\end{array}\right\}\text{Add}$$

The Sum is the Logarith. 2.153510 of $142{,}4 = U$.

Viz. 142 *l.* 8 *s.* will be the Rent required, if it be paid but once a Year; which is juſt 25 Years Value.

But if the Rent muſt be paid Half-yearly, or Quarterly, &c.

Then { *Inſtead of multiplying the Sum propoſed to be laid out in the Purchaſe, with* $R - 1$ (as above) *you muſt take ſuch a part of the Log. of* R (*viz.* of the Amount of 1 *l.*) *as the Times of Payment require, and the Number anſwering to that part being made leſs by* 1, *muſt be the Multiplicator inſtead of* $R - 1$.

Take the ſame Example in Halfyearly Rents.

That is, let there be given $P = 3560$ and $R = 1{,}04$; to find U, the Half-yearly Rent.

Firſt $R = 1{,}04$ the $\frac{1}{2}$ of its Log. is 0.008516 whoſe Number is 1,0198 leſs 1, is 0.0198 the Multiplicat.

Then $3560 \times 0{,}0198 = 70{,}488$, *viz.* 70 *l.* 9 *s.* 9 *d.* $= U$; the Half-yearly Rent.

Or, having found the New Multiplicator, as above;

$$\left.\begin{array}{lll}\text{Then } P = 3560 & \text{its Log. is} & 3.551450\\ \text{Multiplicat. } 0{,}0198 & \text{its Log.} & 8.296665\end{array}\right\}\text{Add}$$

The Sum is the Logarith. 1.848115 of $70{,}488 = U$;

Viz. 70 *l.* 9 *s.* 9 *d.* is the Half-yearly Rent, as before.

Example 3. For Quarterly Rents.

Suppoſe the ſame Sum of 3560 l. *were propoſed to be laid out in the Purchaſe of a Freehold Eſtate; and it were required to find what Quarterly Rent it would buy, at the Rate of Four* per Cent. &c.

Here is alſo given $P = 3560$ and $R = 1,04$; to find U, the Quarterly Rent. Thus,

Firſt $R = 1,04$ the $\frac{1}{4}$ of its Log. is 0.004258 whoſe Number is 1,00986 and $1,00986 - 1 = 0,00986$ the New Multiplicator.

Then $3560 \times 0,00986$ gives 35,1; *viz.* 35 *l.* 2 *s.* $= U$, the Rent to be paid every Quarter; which is leſs than the $\frac{1}{4}$ of the Annual Rent by 10 *s.* *per* Quarter.

Whence it plainly appears, that the leſs the Intervals of the Times are, betwixt the Payments of the Rents, the more valuable the Purchaſe is; *& vice verſa.*

Caſe 3. The Annual Rent of any Freehold Eſtate, and the Sum that is paid for it, being known; to find what Rate of Intereſt *per Cent.* is allow'd to the Purchaſer.

Rule. { *Divide the Annual Rent by the Sum that is paid for the Purchaſe, and the Quotient will ſhew the Ratio of the Rate of Intereſt* per Cent.

Example.

Suppoſe a Freehold Eſtate of 250 l. per Annum, *coſt* 4166 l. 13 s. 4 d. *what Rate of Intereſt* per Cent. *is allow'd the Purchaſer?*

In this Example there is given $U = 250$, $P = 4166,667$, to find R, or rather $R - 1$, the Ratio of the Rate.

Thus 4166,667) 250,000(0.06 $= R - 1$. Then it will be, As 1 : is to 0,06 : : So is 100 : to 6, the Rate of Intereſt *per Cent.* as was required.

Or by Logarithms; thus,

Firſt $U = 250$ its Log. is 2.397940 } Subſtract
And $P = 4166,667$ its Log. 3.619789 }

The Difference is the Log. $\bar{8}.778151$ of $0,06 = R - 1$
Or to the laſt Log. add this 2.000000 of 100.

The Sum will be the Log. 0.778151 of 6 the Rate of Intereſt *per Cent.* as before.

Thus far concerning ſuch Queſtions that are about *Intereſt*, *Annuities*, or *Leaſes*, &c. as are limited by any aſſign'd Time, and Real Eſtates that are purchaſed to continue for ever; and 'tis only ſuch that can be computed by any certain Rules: However, it may not (I preſume) be unacceptable to the Reader, if I proceed a little further, and inſert a brief Account of ſuch Helps to eſtimate the Values of Annuities, or Leaſes for Lives, as are propoſed by two very Ingenious Perſons.

Sect. 2. *How to eſtimate the Value of Annuities, or Leaſes for Lives.*

THE Way generally uſed in buying of Annuities, or letting of Leaſes for Lives, is only by an imaginary Valuation, grounded upon Cuſtom, and not upon any Conſideration that is had to the Age of the Perſons whoſe Lives are to be inſerted in the Leaſe, *&c.* 'Tis true indeed, that there can be no certain Rules preſcribed for their true Valuation, becauſe the Lives of all Mankind are uncertain; and 'tis poſſible, and daily ſeen, that a young Man may die before one of a greater Age: But yet there is a greater probability of a young Man's living longer than an old one; and not only ſo, but there's a proportional Likelihood of the Length of Mens Lives, according to their different Degrees of Age. Which being duly conſidered, muſt needs be found of good uſe in eſtimating the Values of Annuities or Leaſes for Lives, much better than by a mere Gueſſing only, as uſual: And that ſuch a proportional Likelihood is worth the Conſideration, will appear from what follows.

1. Sir *William Petty*, in a Diſcourſe of his, made before the Royal Society (*Anno* 1674.) concerning the Uſe of *Duplicate Proportion*, doth amongſt other things apply it to the Life of Man, and its Duration, thus:

It is found by Experience (*ſaith he*) that there are more Perſons living of between 16 and 26 Years old, than of any other Age or Decad of Years in the whole Life of Man, (*which* David *and Experience ſay to be between* 70 *and* 80 *Years.*) The Reaſons whereof are not abſtruſe; *viz.* becauſe thoſe of 16 have paſs'd the Danger of *Teeth*, *Convulſions*, *Worms*, *Rickets*, *Meazles*, and *Small-Pox* for the moſt part: And that thoſe of 26 are ſcarce come to the *Gout*, *Stone*, *Dropſy*, *Palſies*, *Lethargies*, *Apoplexies*, and other Infirmities of Old Age. Now whether theſe be ſufficient Reaſons, is not the preſent Enquiry; but taking the aforementioned Aſſertions to be true, I ſay, that the Roots of every Number of Men's Ages under 16 (*whoſe*

Root is 4) compared with the ſaid Number 4, doth ſhew the Proportion of the Likelihood of ſuch Mens reaching to 70 (*or* 80) Years of Age.

As for example: 'Tis four times more likely, that one of 16 Years old ſhould live to 70, than a new-born Babe. 'Tis three times more likely, that one of nine Years old ſhould attain the ſaid Age of 70, than the ſaid Infant. Moreover, 'tis twice as likely, that one of 16 ſhould reach that Age, as that one of 4 Years old ſhould do it; and one third more likely, than for one of 9.

On the other hand, 'Tis five to four, that one of 25 Years old will die before one of 16; and ſix to five, that one of 36 will die before one of 25; and three to two, that the ſame Perſon of 36 ſhall die before him of 16: and ſo forward, according to the Roots of any other Year of the declining Age, compared with a Number between 4 and 5 (viz. *with* 4,58) which is the Root of 21, the moſt hopeful Year for Longevity, as the Mean between 16 and 26; and is the Year of Perfection, according to the Senſe of *our Law*, and the Age for whoſe Life a *Leaſe* is moſt valuable.

To prove all which, I can produce the Accompts of every Man, Woman, and Child within a certain Pariſh of 330 Souls; all which particular Ages being added together, and the Sum divided by the whole Number of Souls, made the Quotient between 15 and 16: which I call the Age of that Pariſh, or Numerous *Index* of Longevity there.

Thus you have a Learned Gentleman's Opinion concerning the Likelihood of the Length of Mens Lives, according to the Rules of *Duplicate Proportion*; which was a very ingenious Thought of his: But I muſt beg pardon, that I cannot agree with him in that part of it, which aſſerts, That 21 Years is the Age for whoſe Life a Leaſe is the moſt valuable. For although 'tis true, that according to our Law, a Man is ſaid to be then at his Perfect or Full Age, as to the Enjoyment of an Eſtate, or Managing his Affairs without a Guardian, *&c.* yet I ſhould rather adhere to the Cloſe of his Diſcourſe, wherein he ſays, That he found, that if the Sum of all the Ages of the 330 Souls (*in a certain Pariſh*) being divided by their Numbers, made the Quotient between 15 and 16. Whence I take 16 to be the Age, for whoſe Life a Leaſe is the moſt valuable;

luable; and upon that Suppoſition I have calculated the following Table, according to the aforeſaid Rules of *Duplicate Proportion.*

Ages	*Years Purch.*	*Ages*	*Years Pur.*	*Ages*	*Years Pur.*
1	2,50	26	7,20	51	5,04
6	5,51	31	6,46	56	4,81
11	7,46	36	6,00	61	4,60
16	9,00	41	5,62	66	4,43
21	7,85	46	5,31	71	4,27

This Table ſhews, by Inſpection, the Value of every five Years of any ſingle Life, from the Birth to 71 Years old. Suppoſing that any Annuity, or Leaſe, &c. is really worth nine Years Purchaſe for one Life; which is according to the Rate that the Annuities ſettled by a late Act of Parliament for Lives, were valued at, and from thence the reſt are computed.

As for inſtance: An Annuity, or Leaſe of 100 *l. per Ann.* is by this Table worth 900 *l.* (*viz.* 9 × 100) for the Life of one that is about 16 Years old; and 'tis worth but 600 *l.* for the Life of one that is 36 Years old, and but 250 *l.* for the Life of a Child of a Year old: And ſo on for any other propoſed Annuity, or Leaſe, and Age of the Perſon for whoſe Life it is to be purchaſed; which is ſo eaſy to apprehend, that I need ſay no more of its Uſes, but proceed to the next.

2. The Ingenious and Great Mathematician, Dr. *Edmund Halley*, *Savilian Profeſſor* of *Geometry* in the *Univerſity* of *Oxford*, hath (in the *Philoſophical Tranſactions*, *Numb.* 196.) given us a moſt excellent *Eſſay* to eſtimate the Degrees of the *Mortality* of Mankind, which he deduced with a great deal of Art and Labour, from curious Tables of the *Births* and *Funerals* that were in *Breſlaw*, the Capital City of the Province of *Sileſia* in *Germany*, for five Years ſucceſſively, *viz.* from the Year 1687, to 1691 incluſive, drawn up Monthly by one Dr. *Newman* of that City, and communicated to the *Royal Society* here; from whence Dr. *Halley* hath form'd the following Table: which is, without doubt, very carefully and exactly done, and does give a more perfect Account of the State and

Condition of Mankind in reſpect of their *Chances* of *Mortality* at all *Ages*, and conſequently how to eſtimate the *Values* of *Annuities*, or *Leaſes* for Lives, better than by any other Method ever propoſed before it. The *Table* does ſhew the *Number* of *Perſons* that were living in their reſpective *Ages Current* annexed thereto, as follows.

Ages Cur.	*Perſons Living.*	*Ages Cur.*	*Perſons Living.*	*Ages Cur.*	*Perſ. Liv.*	*Ages Cur.*	*Perſ. Liv.*
1	1000	22	586	43	417	64	202
2	855	23	579	44	407	65	192
3	798	24	573	45	397	66	182
4	760	25	567	46	387	67	172
5	732	26	560	47	377	68	162
6	710	27	553	48	367	69	152
7	692	28	546	49	357	70	142
8	680	29	539	50	346	71	131
9	670	30	531	51	335	72	120
10	661	31	523	52	324	73	109
11	653	32	515	53	313	74	98
12	646	33	507	54	302	75	88
13	640	34	499	55	292	76	78
14	634	35	490	56	282	77	68
15	628	36	481	57	272	78	58
16	622	37	472	58	262	79	49
17	616	38	463	59	252	80	41
18	610	39	454	60	242	81	34
19	604	40	445	61	232	82	28
20	598	41	436	62	222	83	23
21	592	42	427	63	212	84	20

This Table may be applied to very many Uſes, but I ſhall only inſert what may ſuffice for the preſent Purpoſe.

1. The firſt Uſe is to ſhew the differing Degrees of *Mortality*, or rather *Vitality* in all Ages; for if the Number of Perſons of any Age remaining after one Year, be *divided* by the *Difference* between that and the *Number* of the Age propoſed, it ſhews the *Odds* that there is, that a Perſon of that Age does not *die* in one Year.

As for inſtance, a Perſon of 25 Years of Age has the *Odds* of 560 to 7 (*viz.* 80 to 1) that he does not *die* in a

Year; becauſe that of 567 Perſons living of 25 Years of Age, there do die no more than 7 in a Year, leaving 560 of 26 Years old. Or,

2. If it be required to find the odds, that any Perſon does not die before he attain to any propoſed Age; Then

Take the Number of the remaining Perſons of the Age propoſed, and divide it by the Difference between it and the Number of thoſe of the Age of the Party propoſed; and that ſhews the odds there is between the Chances of the Party's living or dying.

As for Inſtance: What is the odds that a Man of 40 may live 7 Years? Take the Number of Perſons of 47 Years, which in the Table is 377, and ſubſtract it from the Number of Perſons of 40 Years, which is 445, and the Difference is 68, *viz.* 445 — 377 = 68, which ſhews that the Perſons dying in the 7 Years are 68, and that 'tis 377 to 68, or 5½ to 1, that a Man of 40 does live 7 Years: And the like for any other Number of Years.

3. And if it be required to find at what Number of Years, it is an even lay, that a Perſon of any propoſed Age ſhall die, this Table readily performs it: For if the Number of Perſons living of the Age propoſed be halv'd, it will be found by the Table at what Year the ſaid Number is reduced to half by Mortality; and that is the Age, to which it is an even Wager, that a Perſon of the Age propſed ſhall arrive to before he die.

As for inſtance, a Perſon of 30 Years of Age is propoſed; the Number of that Age is 531, the half of it it 265; which Number I find to be between 57 and 58 Years: So that a Man of 30 may reaſonably expect to live between 27 and 28 Years.

4. By what has been ſaid, the Price of Inſurance upon Lives ought to be regulated, and the Difference is diſcovered between the Price of inſuring the Life of a Man of 20 and one of 50 Years old.

For example; It being 100 to 1, that a Man of 20 dies not in a Year, and but 38 to 1, for a Man of 50 Years of Age.

5. And upon theſe Proportions depend the Valuation of Annuities for Lives: For it is plain, that the Purchaſer ought to pay only for ſuch a Part of the Annuity, as he hath Chances that he is living; and this ought to be computed yearly, and the Sum of all thoſe yearly Values be-

ing added together, will be the Value of the Annuity for the Life of the Perſon propoſed. Now the preſent Value of Money payable after any Term of Years, may be eaſily computed by *Caſe* 2. *Page* 162. at any given Rate of Intereſt.

Then

As the Number of Perſons living after that Term of Years, to the Number dead; ſo are the Odds that any one Perſon is alive or dead.

And by conſequence, as the Sum of both, or the Number of Perſons living of the Age firſt propoſed: To the Number remaining after ſo many Years (both being given by the Table) : : *So is the preſent Value of the yearly Sum payable after the Term propoſed: To the Sum which ought to be paid for the Chance the Perſon has to enjoy ſuch an Annuity ſo many Years. And this being repeated for every Year of the Perſon's Life, the Sum of all the preſent Values of thoſe Chances is the true Value of the Annuity.*

Now becauſe the Work of theſe Proportions is ſomewhat troubleſome to perform, the ingenious Author hath been ſo kind as to take the pains (which was not a little) to calculate the following Table; which ſhews the Value of Annuities, or Leaſes, *&c.* (at the rate of 6 *per Cent.*) for every fifth Year of Age to the ſeventieth.

Ages	*Years Purch.*	*Ages*	*Years Pur.*	*Ages*	*Years Pur.*
1	10,28	25	12,27	50	9,21
5	13,40	30	11,72	55	8,51
10	13,44	35	11,12	60	7,60
15	13,33	40	10,57	65	6,54
20	12,78	45	9,91	70	5,32

This Table being of the ſame nature with that in *Pag.* 208, there needs no other Explanation or Example to ſhew its Uſe, than what has been already ſaid about that Table: Only here I muſt again beg leave to give my Opinion about the Difference of the Proportions in the two Tables; which is, that as the Table in *page* 208. may not be thought a ſufficient Guide to be depended upon in eſtimating the length of Mens Lives, &c. becauſe it is only deduced from the bare Rules of Art, *viz.* that of duplicate Proportion; ſo, on the other hand, I doubt the Eſtimates of the Value of any Annuity taken from this Table, will be found too great in this Country, (*viz.* in *England*) which I much fear hath not ſo good or ſalubrious an Air, as that at the City of *Breſlaw*, from whence theſe Calculations are drawn: But if in imitation hereof the Curious in other Cities and large Towns would attempt ſomething of the ſame nature; then, without all doubt, this Method of eſtimating the Probability of the length of Mens Lives would prove the beſt, and become more univerſally uſeful, than can be expected from this one ſingle Inſtance, more eſpecially if ſuch Obſervations were continued for any conſiderable time, as 20 or 30 Years ſucceſſively. And then it would be well worth the time and pains to calculate proper Tables of the Value of Annuities, or Leaſes, &c. both for two, and for three Lives, according to the following Rules; which are deduced from the former Table.

And, Firſt, two Lives being propoſed, their Values may be thus found.

If the Number of Chances of each ſingle Life, found in the Table, be multiplied together, the Product is the Chances of thoſe two Lives; and after any certain Term of Years, the Product of the two remaining Sums will be the Chances that both the Perſons are living: And the Product of the Differences being the Numbers of the Dead of both Ages, will be the Chances that both the Perſons are dead.

Then { *As the Product of the two Numbers in the Table for the two Ages propoſed: Is to the Difference between that Product and the Product of the two Numbers of the Perſons deceaſed in any given Space of Time :: So is the Value of a Sum of Money to be paid after that Time: To the Value thereof under the Contingency of Mortality.*

And if three Lives are propoſed, to find the Value of an Annuity during the continuance of thoſe Lives; Then

As the Product of the continual Multiplication of the three Numbers in the Table, anſwering to the Ages propoſed: Is to the Difference of that Product, and the Product of the three Numbers of the Deceaſed of thoſe Ages *in any given Term of Years :: So is the preſent Value of a Sum of Money, to be paid certainly after ſo many Years : To the preſent Value of the ſame Sum to be paid, provided one of thoſe three Perſons be living at the Expiration of that Term of Years.*

Theſe Proportions being yearly repeated, the Sum of all thoſe preſent Values will be the Value of the Annuity granted for thoſe Lives.

Theſe Rules are explain'd at large by their Author, both *Algebraically* by Letters, and by *Geometrical Figures* : And he alſo proceeds on (by the ſame Method) to compute the preſent Value of the Reverſion of any Annuity, or Leaſe, either of one Life after another, or after two Lives, *&c.* The which being not only too long a Diſcourſe to be inſerted in this ſmall *Treatiſe*, but alſo too difficult a Piece of Work for any ordinary Arithmetician to undertake ; I have therefore omitted it, and refer thoſe that are curious, and deſire further Satisfaction therein, to the aforemention'd *Philoſ. Tranſ. Numb.* 196. and ſhall only add this ſerious Obſervation : *viz.* How unjuſtly we generally repine at the Shortneſs of our Lives, and think ourſelves wrong'd if we attain not to Old Age : whereas it appears, that the one half of thoſe that are born, do die in ſeventeen Years Time. For by the aforeſaid Bills of *Mortality* at *Breſlaw*, it was found, that 1238 were in that time reduced to 616. So that inſtead of murmuring at what we call a *Short Life*, we ought with Patience and Unconcern to ſubmit to that Diſſolution, which is the neceſſary Condition of our periſhable Materials, and of our nice and frail Structure and Compoſition ; and to account it as a great Bleſſing that we have ſurvived (perhaps by many Years) that Period of Life, whereat the one half of the whole Race of Mankind does not arrive.

Having now gone through all the general Caſes of Intereſt, and Annuities, *&c.* I deſign'd to have concluded here ; but becauſe the Buſineſs of *Rebate* or *Diſcompt* of Money paid before the Time it becomes due, comes often into practice upon ſeveral Occaſions ; and being but juſt touch'd upon in *page* 136, and 162. altho' even what is there done, being duly conſider'd, might be ſufficient : yet, leſt I ſhould be thought too ſhort or remiſs in ſo uſeful a Part of Intereſt as that is, it may be convenient to proceed a little further, and lay down particular Rules for that Purpoſe.

CHAP. V.

To find the Rebate and Diſcompt of any propoſed Sum, &c. and the true Equated Time of ſeveral Payments; either at Simple, or Compound Intereſt.

Sect. 1. *To find the true Diſcompt of any Sum, at any Rate of Simple Intereſt.*

SUppoſe S = the Sum propoſed to be diſcompted for;
T = the Interval of Time it becomes due;
R = the Ratio of the Rate of Intereſt *per Cent.*
And D = the Diſcompt or Rebate ſought.

Then it will be $$\frac{TRS}{TR+1}=D.$$

Which in Words gives this following Rule.

Rule. *Firſt multiply the given Time with the Ratio of the Rate of Intereſt, and to their Product add* 1, *that Sum will be the Diviſor.*

Next multiply the firſt Product and the Sum that is to be diſcompted for, and that Product will be the Dividend; the Quotient ariſing from thence will ſhew the Diſcompt requir'd.

Example.

Let it be required to find what Diſcompt ought to be allowed for 3560 l. *if it be paid* 273 *Days before it becomes due, at the Rate of* 6 per Cent. per Annum, *Simple Intereſt?*

Here is given S = 3560, T = 0,74794, (found by the Table in *pag.* 141.) and R = 0,06; to find D.

First 0,74794 × 0,06 = 0,044876.

And 0,044876 + 1, is 1,044876 for the Diviſor.

Next 0,044876 × 3560 = 159,75856 the Dividend,

Then 1,044876) 159,75856 (152,897 = D.

That is 152 *l.* 18 *s. fere*, will be the Diſcompt requir'd.

The ſame perform'd by *Logarithms.*

First, $T = 0{,}74794$ its Log. $\bar{9}.873867$ }
And $R = 0{,}06$ its Log. is $\bar{8}.778151$ } Add.

The Sum is the Log. $\bar{8}.652018$ of $0{,}044876$
Next $S = 3560$ its Log. is 3.551450 add to the laſt
Sum 2.203468 }
$0{,}044876 + 1 = 1{,}044876$ Log. 0.019066 } Subſtract

There remains the Logar. 2.184402 of 152,897; that is, 152 *l.* 18 *s.* as above; which being ſubſtracted from 3560 *l.* the propoſed Sum, there will remain 3407 *l.* 2 *s.* the Sum to be paid in ready Money; as may be eaſily proved by making it a Principal, and then finding what it would amount to in 273 Days at 6 *per Cent.* which, by *Caſe* 1, *page* 134. will be juſt 3560 *l.* Conſequently the Diſcompt is truly found.

Secondly, To find the true Rebate or Diſcompt of any Sum, at any given Rate of Compound Intereſt.

Let us ſuppoſe S, t, and D, to repreſent the ſame Parts of the Queſtion, as before; and R, to denote the Amount of 1 *l.* &c. as in *page* 161.

$$\text{Then will } \left\{ \frac{SR^t - S}{R^t} = D. \right.$$

And from this Equation is deduced the following Rule.

Rule. { *Multiply the Logarithm of* R *the Amount of* 1 l. *at the given Rate of Intereſt, with the Time, and the Number which belongs to their Product will be the Diviſor.*

From that Number ſubſtract 1, *then multiply the Remainder with the propoſed Sum that is to be diſcompted for, and that Product will be the Dividend; the Quotient ariſing from thence will ſhew the Diſcompt required.*

Example.

Suppoſe it were required to find what Diſcompt or Rebate muſt be allow'd for the Payment of 956 l. 10 s. *nine Months* (viz. ¾ *of a Year*) *before it becomes due, at the Rate of Five and a Half* per Cent. *Compound Intereſt.*

In this Queſtion there is given $S = 956{,}5$, $t = 0{,}75$, and $R = 1{,}055$; to find D.

$$\left.\begin{array}{r}\text{Firſt, } R = 1{,}055 \text{ its Log. is } 0.023252\\ \text{And } \underline{t = \quad 0{,}75}\end{array}\right\}\text{Multiply}$$

The Product is the Log. 0.017439 of 1,04097 the Diviſor: And 1,04097 — 1 = 0,04097 will be the Multiplicator: And 956,5 × 0,04097 = 39,1878 the Dividend. Then 1,04097) 39,1878 (37,645 = *D*; *viz.* 37 *l.* 12 *s.* 10 ¾ *d.* is the Diſcompt required.

Or, having firſt found the Diviſor, and by it the Multiplicator, as above; then the reſt of the Work is very eaſily perform'd by Logarithms.

$$\left.\begin{array}{r}\text{Thus } S = 956{,}5 \text{ its Logar. is } 2.980685\\ \text{Multiplicator } 0{,}04097 \text{ its Log. } \underline{\bar{8}.612466}\end{array}\right\}\text{Add}$$

$$\left.\begin{array}{r}\text{Sum } 1.593151\\ \text{Diviſor is } 1{,}04097 \text{ its Log. is } \underline{0.017439}\end{array}\right\}\text{Subſtract}$$

There remains the Logarith. 1.575712 of 37,645, *viz.* 37 *l.* 12 *s.* 10 ¾ *d.* is the Diſcompt required, as above; which being taken from the 956 *l.* 10 *s.* leaves 918 *l.* 17 *s.* 1 ¼ *d.* the Sum which is to be paid in ready Money; as may be tried, by making it a Principal, and then finding what it would amount to in ¾ of a Year at the Rate of Five and a Half *per Cent.* the which, by *Caſe* 1. *pag.* 161. will be found to be juſt 956 *l.* 10 *s.* Therefore, *&c.*

If theſe two Rules, and their Examples, be a little conſider'd, I preſume it will be very eaſy to conceive how to find the true Diſcompt or Rebate of any one propoſed Sum of Money, due at the end of any given Interval of Time, and any given Rate, either of Simple or Compound Intereſt, according as the Queſtion is propoſed.

And if it be required to find the whole Diſcompt of ſeveral Sums, due at the end of ſeveral different Intervals of Time, it is but computing them at ſo many ſeveral Operations; and then the Sum of all thoſe particular Diſcompts being taken from the total Sum of all the Debts, will leave the true Diſcompt required.

As for inſtance;

Suppoſe A *were indebted to* B 750 l. *to be paid at three ſeveral Payments, in this manner;* viz. 250 l. *at the end of one Year and a half,* 100 l. *to be paid at the end of two Years, and* 400 l. *at the end of four Years; the Queſtion is, To find how much of the* 750 l. B *ought to rebate or*

Diſcompt, at Six per Cent. per Annum *Simple Intereſt, to have his whole Debt diſcharged by* A *in preſent Money?*

According to the *Data* in this Queſtion, the particular Diſcompts (found by the firſt Rule in this Section) will be theſe following:

The Diſcompt of { 250 *l.* for 1,5 Year is 20,6423 } Add
{ 100 *l.* for 2 Years is 10,7142 }
{ 400 *l.* for 4 Years is 77,4193 }

Conſequently the whole Diſcompt is 108,7758

Then if from 750 *l.* the whole Debt, there be ſubſtracted ..... 108,7758 the whole Diſcompt, there will remain .. 641,2242 = 641 *l.* 4 *s.* 5¾ *d.* which is the true Sum that *A* muſt give to *B*, in preſent Money, to be diſcharg'd of his Debt.

Again, Suppoſe the ſame things given as above, and let it be requir'd to find the particular Diſcompts, *&c.* at Compound Intereſt.

Then, by the ſecond Rule in this Section, the Work will ſtand thus:

The Diſcompt of { 250 *l.* for 1,5 Year is 20,9238 } Add
{ 100 *l.* for 2 Years is 11,0003 }
{ 400 *l.* for 4 Years is 83,1633 }

Sum is 115,0874

Then 115,0874 taken from 750 *l.* leaves 634,9126, *viz.* 634 *l.* 18 *s.* 3 *d.* is now the Sum which *A* is to give *B*, in preſent Money, to be clear of his Debt.

This is ſo plainly to be underſtood, that I preſume it's needleſs to ſay any more in Proof of it, than what has been already ſaid in *Page* 215, and 216.

And from hence naturally flows the following Method of finding the true Equated Time, wherein ſeveral Sums, due at ſeveral Intervals of Time, may be paid at one entire Payment, without any Loſs, either to the Creditor or Debtor.

Sect. 2. *The Equated Time of Payments truly determin'd.*

THE uſual Rule laid down in divers Treatiſes of Arithmetick and Intereſt, *&c.* for finding of an Equated Time for the Payment of ſeveral Sums of Money due at the End of unequal Intervals of Time, is to this Effect.

Rule. *Multiply every ſingle Sum of Money with the Time it becomes due; and divide the Sum of thoſe Products by the Total Debt; and the Quotient will be the true Time* (ſay they) *at which all the Money ought to be paid.*

I ſhall paſs over all the Arguments that are made uſe of *pro* and *con*, by Mr. *John Kerſey*, in his Book of Arithmetick, and by Sir *Samuel Morland*, in his Doctrine of Intereſt, and other Authors, about the Erroneouſneſs of this Rule, as alſo the Rules they lay down inſtead of it; and ſhall only proceed to ſhew how the true Equated Time may be found, from what hath been already done and proved.

When any Number of Payments are propoſed to be paid off at one entire Sum; then, in order to determine the Equated Time for the Payment of that Sum, you muſt firſt find the particular Diſcompts of all the propoſed Payments, whoſe Times are to be Equated, according to the given Intervals of thoſe Payments, at any Rate, either of Simple, or Compound Intereſt, as ſhall be agreed on by the Parties concerned; and by thoſe Diſcompts find the preſent Sum that would clear the Debt, if it were immediately paid; (*as above.*)

Then *If the Sum of all thoſe particular Diſcompts, be divided by the Product of the preſent Sum multiplied with the Ratio of the ſame Rate of Intereſt by which thoſe Diſcompts were computed, the Quotient will ſhew the true Equated Time required at Simple Intereſt.*

For an Example of this Rule;

Let it be requir'd to find the true Equated Time, wherein the aforeſaid three Sums, due from A *to* B, (as in the laſt Inſtance) *may be ſafely paid without Loſs to either*, &c.

There the Sum of all the particular Diſcompts at Six *per Cent.* Simple Intereſt, is found to be 108,7758 and 750 — 108,7758 = 641,2242 the preſent Sum, if it were to be immediately paid.

Then, by the Rule above, 641,2242 × 0,06 = 38,47345 will be the Diviſor; and 108,7758 the Dividend. Then 38,47345) 108,7758 (2,8273 is the Quotient; that is, 2 Years and 302 Days will be the Time when *A* may pay unto *B* his whole Debt of 750 *l.* without any Loſs to either of them.

The ſame found by Logarithms.

Sum of *Diſcompts* 108.7758 its Log. 2.036544
Preſent Worth is 641,2242 its Log. 2.807010 } Add
Ratio of the *Rate* is 0,06 its Log. 8.778151 }
From the firſt Log. ſubſtr. this Log. 1.585161 Sum.

There remains the Logarithm 0.451383 of 2.8273 *viz.* 2 Years and 302 Days will be the true Equated Time required by the Queſtion; as before.

For if the Diſcompts be (truly found, then it cannot be denied, but that the Preſent Worth of the whole Debt may by them be truly found: And, I ſay, if that Preſent Worth be made a Principal, and the Equated Time (*as here found*) be made the Time of that Principal's being forborn at the ſame Rate of Intereſt, it will be found (*per Caſe* 1. *pag.* 135.) to amount to juſt the whole Debt; therefore the Equated Time is here truly found.

And if it be required to find the Equated Time at any propoſed Rate of Compound Intereſt, you muſt proceed in the ſame manner to find all the particular Diſcompts of the given Sums, and by them the preſent Sum that would clear the Debt, if it were immediately paid; as in *pag.* 215.

Then { *If the Log. of the preſent Sum be taken from the Log. of the Total Sum of all the Debts, and the Remainder be divided by the Log. of R, the Amount of* 1 l. *at the ſame Rate of Intereſt by which thoſe Diſcompts were computed; the Quotient will ſhew the Equated Time requir'd.*

As for Example:

Suppoſe it were requir'd to find the Equated Time at Compound Intereſt, wherein the 750 l. *due from* A *to* B, *in the ſame manner as before, may be paid at one entire Payment, without Loſs,* &c.

Then the Sum of all the particular Diſcompts, found at 6 *per Cent.* Compound Intereſt, is 115,0874
And 750 − 115,0874 = 634,9126 is the preſent Sum that would diſcharge the Debt, if it were immediately paid, as in *page* 217. Then, by the Rule above, it will be

750 its Logarithm is . . . . 2.875061 } Subſtr.
Preſent Sum 634,9126 its Logar. is 2.802714 }
R = 1,36 its Logar. is 0.025306) 0.072347 (2,8588
That is, 2 Years, and 313 Days, is here found to be the

Equa-